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# A lattice Boltzmann model for condensation and freezing of dry saturated vapor about a cryogenic spot on an inclined hydrophobic surface

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ABSTRACT

A triple phase-change lattice Boltzmann model, based on a coupling of existing vapor-liquid phase-change model and enthalpy-based liquid-solid phase-change model, is proposed in this paper. Based on this novel model, 3D simulations for transient condensation of a dry saturated vapor and subsequent freezing at a circular cryogenic spot of an inclined hydrophobic surface are carried out. The dynamic morphologies of the droplet and ice are presented. Simulated results show that ice adjacent to the cryogenic spot is formed inside a droplet, and the droplet subsequently moves down along the vertical surface under action of gravity. Effects of droplet's dynamic behaviors on the growth of ice and averaged heat flux are presented. Temperature distributions in the ice as well as in the water and steam on the middle cross section and local heat flux at the wall are illustrated. Simulations of the triple phase-change phenomena under different tilt angles show that the ice is elongated in the direction of gravity. The wettability of the surface is shown to have important influence on the morphology of droplet but its effect on the growth of ice is small.



## 1. Introduction

Condensation and subsequent freezing of vapor on surfaces at temperature below freezing point have important applications in air-conditioning, refrigeration, aircrafts fight safety and power engineering industries. In the past several decades, much experimental work has been carried out to investigate transition from vapor to liquid, and finally to solid as the wall is decreased to cryogenic temperature [1].

Wu et al. [2] performed an experiment on condensation and subsequent freezing of steam on a horizontal surface, and studied the complete process of triple phase-changes, including droplet nucleation and growth as well as freezing of droplets. Li et al. [3] designed an experiment to study ice nucleation of micro-droplets on cold horizontal surfaces having different contact angles. Boreyko and Collier [4] investigated condensation and subsequent freezing of water on horizontal superhydrophobic surfaces, and reported that some droplets jumped off the surface before freezing occurred. By conducting similar experiments, Oberli et al. [5] concluded that rapid kinetic freezing of droplets on superhydrophobic surfaces was initiated by self-seeding from a freezing front.

Experimental investigations of phase-change phenomena, however, have many limitations because detailed information on fluid flow and thermal fields cannot be measured. On the other hand, numerical simulation can offer an important tool to study the physical process involved. Recently, Gong and Cheng [6] developed an improved liquid-vapor phase-change lattice Boltzmann method based on Hazi and Markus's work [7], and obtained boiling curves numerically for the first time [8]. Using the same phase-change LB model, Liu and Cheng simulated laminar film condensation on a vertical hydrophilic subcooled surface [9] as well as dropwise condensation on a vertical hydrophobic surface with a subcooled cold spot [10]. Most recently, the same model was used by Li and Cheng [11] to study transition from dropwise to filmwise condensation on a horizontal downward-facing subcooled surface and obtained condensation curves numerically. Meanwhile, lattice Boltzmann methods have been developed for solid-liquid phase-change as well [12–18]. Jiaung et al. [12] were among the first to develop a lattice Boltzmann model in terms of enthalpy to simulate heat conduction problems with phase change. Subsequently, Chatterjee and co-workers [14,15] proposed a series of enthalpy-based lattice Boltzmann models for solid-liquid phase-change with fluid flow taken into consideration. Huang et al. [16] developed a new approach to treat the latent-heat source term by modifying the equilibrium distribution function. More recently, Su and Davidson

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**Nomenclature**

| | |
|---|---|
| $B$ | weighting function |
| $c_p$ | specific heat at constant pressure (J kg$^{-1}$ K$^{-1}$) |
| $c_s$ | lattice sound speed (m/s) |
| $c_{solid}$ | specific heat capacity of solid (J kg$^{-1}$ K$^{-1}$) |
| $c_v$ | specific heat at constant volume (J kg$^{-1}$ K$^{-1}$) |
| $D$ | diameter of cryogenic spot (m) |
| $f_s$ | volume fraction of solid phase |
| $Fo$ | Fourier number |
| $\mathbf{F}$ | total force (N) |
| $\mathbf{F}_g$ | gravitational force (N) |
| $\mathbf{F}_{int}$ | inter-particle interaction force (N) |
| $\mathbf{F}_s$ | fluid-wall interaction force (N) |
| $\mathbf{g}$ | gravitational acceleration (m/s$^2$) |
| $H$ | enthalpy (J kg$^{-1}$) |
| $\Delta H_{ls}$ | latent heat of solid-liquid phase-change (J/kg) |
| $l_0$ | characteristic length (m) |
| $q$ | heat flux (W/m$^2$) |
| $q_0$ | reference heat flux (W/m$^2$) |
| $T$ | temperature |
| $T^*$ | dimensionless temperature |
| $\mathbf{u}$, $\mathbf{U}$ | velocity vector (m/s) |
| $V_{ice}$ | volume of ice |
| $V^*_{ice}$ | dimensionless volume of ice |

*Greek symbol*

| | |
|---|---|
| $\theta$ | static contact angle (°) |
| $\varphi$ | tilt angle of flat plate (°) |
| $\Psi$ | effective mass |
| $\Phi_{vl}$ | source term for vapor-liquid phase-change |
| $\Phi_{ls}$ | source term for liquid-solid phase-change |
| $\tau$ | relaxation time |
| $\Omega^s$ | additional collision term |
| $\nu$ | kinematic viscosity (m$^2$/s) |
| $\chi$ | thermal diffusivity (m$^2$/s) |

*Subscripts or superscripts*

| | |
|---|---|
| ave | average |
| eq | equilibrium |
| l | liquid |
| s | solid |
| sat | saturation |
| v | vapor |
| w | wall |

[17] developed a timestep adjustable non-dimensional lattice Boltzmann model for solid-liquid phase-change using a scaling analysis based on the mesoscopic length and velocity scales. Zhao et al. [18] used an enthalpy-based lattice Boltzmann model to investigate freezing of a free falling droplet in a cold environment with forced convection taken into consideration.

It is relevant to point out that the freezing process is usually preceded by condensation heat transfer. However, previous lattice Boltzmann models [12–18] developed for freezing have not taken the vapor-liquid phase-change into account. In this paper, we develop a novel enthalpy-based lattice Boltzmann model coupling with Gong-Cheng's improved liquid-vapor phase-change model [6] for simulation of triple phase-change heat transfer phenomena. In this model, the liquid-vapor has a diffuse-interface which is separated based on pseudo-potential model [19]. The solid-liquid interface is traced by updated enthalpy, and the velocity condition on solid-liquid interface is treated by the immersed moving boundary scheme [20]. Using this novel triple phase-change model, we simulated the 1D transient problem of condensation and freezing of a dry saturated vapor next to a wall of infinite extent as the wall is suddenly drops to a cryogenic temperature. The 3D transient problem of a dry saturated vapor and subsequent freezing about a circular cryogenic spot of an inclined hydrophobic flat plate is simulated as well.

## 2. Description of the triple phase-change simulation model

In this section, we will present a novel triple phase-change lattice Boltzmann model, which couples Gong-Cheng's improved liquid-vapor phase-change model [6] and an enthalpy-based solid-liquid phase-change model by Jiaung et al. [12]. This double distribution function triple phase-change lattice Boltzmann model includes two distribution functions, one for the multiphase flow field and another for the temperature field.

### 2.1. Lattice Boltzmann equation for multiphase flow

Pseudo-potential model [19] is adopted to tracking and separating the liquid-vapor interface, and the immersed moving boundary scheme [16] is used to simulate the moving solid-liquid interface. The evolution equation for the density distribution function, including solid, liquid and vapor, is expressed in the following form [21]:

$$f_i(\mathbf{x}+\mathbf{e}_i\delta t, t+\delta t) - f_i(\mathbf{x},t) = -\frac{(1-B)}{\tau_f}(f_i(\mathbf{x},t) - f_i^{eq}(\mathbf{x},t)) + B\Omega_i^s + (1-B)\Delta f_i(\mathbf{x},t) \quad (1)$$

where $f_i$ is the density distribution function at position $\mathbf{x}$ and time $t$, $\delta t$ is time step, $\mathbf{e}_i$ is discrete particle velocity in $i$th direction. $\tau_f$ is relaxation time, related to the kinematic viscosity, calculated by:

$$\tau_f = \frac{\upsilon}{c_s^2\delta t} + 0.5 \quad (2)$$

$B$ is a weighting function, which depends on the volume fraction of solid $f_s$ and the relaxation time $\tau_f$ [21]:

$$B = \frac{f_s(\tau_f - 0.5)}{0.5 - f_s + \tau_f} \quad (3)$$

where the volume fraction of solid $f_s$ is defined as:

$$f_s = \begin{cases} 0, & H > H_l \\ \frac{H-H_s}{H_l-H_s}, & H_s < H < H_l \\ 1, & H < H_s \end{cases} \quad (4)$$

with $H_l$ and $H_s$ being enthalpies of liquidus and solidus, respectively. Eq. (4) shows that $f_s = 1$ for ice, $f_s = 0$ for liquid and vapor and consequently $B = 0$, and $0 < f_s < 1$ for the mushy zone.

$f_i^{eq}(\mathbf{x},t)$ is the corresponding equilibrium distribution function given by:

$$f_i^{eq}(\mathbf{x},t) = \omega_i\rho\left[1 + \frac{\mathbf{e}_i\cdot\mathbf{u}}{c_s^2} + \frac{(\mathbf{e}_i\cdot\mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2}\right] \quad (5)$$

where $\omega_i$ are weighting coefficients, $c_s$ is the lattice sound speed.$\Omega_i^s$ is an additional collision term which bounces back the non-equilibrium part of the distribution function, calculated as:

$$\Omega_i^s = f_{\bar{i}}(\mathbf{x},t) - f_i(\mathbf{x},t) + f_i^{eq}(\rho,\mathbf{u}_s) - f_{\bar{i}}^{eq}(\rho,\mathbf{u}) \quad (6)$$

where $\mathbf{u}_s$ is the velocity of solid, and $\bar{i}$ is the opposite direction of $i$.