International Journal of Heat and Mass Transfer 114 (2017) 992–1000

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Multilayer system of films heated from above

A.S. Ovcharova

Lavrentyev Institute of Hydrodynamics, Siberian Branch, Russian Academy of Sciences, 15, Acad. Lavrentyev Avenue, Novosibirsk 630090, Russia

ARTICLE INFO

Article history:
Received 11 February 2017
Received in revised form 15 June 2017
Accepted 27 June 2017

Keywords:
Multilayer system
Viscous nonisothermal fluid
Interfaces
Marangoni effect

ABSTRACT

The paper deals with the development of convective flows in a complex system of thin liquid layers occurring under the effect of thermal load. To study this process a mathematical model which is based on the decomposition of complex problem into unitary elements (modules) with some set of rules allowing their linkage with each other was constructed. Each module represents a monotypic local model. The fluid flow and heat transfer in each of these modules are described by the Navier-Stokes and thermal conductivity equations. The boundary conditions required to solve these systems of equations are represented explicitly and written in the form of conservation laws. The numerical analysis of effect of thermal load on the characteristics of the liquid layers movement depending on the Marangoni number is carried out. It is shown that physical and thermophysical properties of liquid layers play the decisive role. It is exactly they control the convective flows in layers and determine the position and shape of interfaces. The results of model problem solutions are presented.

1. Introduction

Study of convective flows and interfaces in a complex system of thin liquid layers is very important due to the significance of problems, which are related to a broad industrial application of thin liquid layers (films). The continuous development of science-intensive technologies significantly expands the range of problems associated with mathematical modeling of the convection processes of viscous heat-conducting liquids. In this regard, the number of publications and even reviews on this subject is so great (and it is increasing every year) that their list is endless. Depending on the set goal, the authors use different approaches to both the description of the processes occurring in thin layers of viscous liquid, and the numerical solution of the arising tasks. Here, the correctness of the selection of an appropriate model describing the phenomenon under study is, of course, of decisive importance.

Mathematical models for describing the processes of mass and heat transfer require correct conditions at the interfaces of liquids. Especially in the case of convective currents of two or more liquid mediums contacting at deformable interfaces. One of the difficulties in solving problems with deformable boundaries is that along with the unknown flow characteristics, it is also necessary to determine the position of the interfaces which change during the motion.

System-based derivation of conditions at the interface between two immiscible liquids under nonisothermal unstable flow was first given in [1]. The modification of these considerations is given in [2]. The purpose of this work, which can be considered as a continuation of the work [3], is the construction of a mathematical model for studying the processes arising in a system consisting of any finite number of thin liquid layers with deformable interfaces. This mathematical model is formulated on the basis of known hypotheses about the physical processes provided by the implementation of conservation laws. It allows us to investigate a wide range of problems associated with the impact of temperature or surfactants on a multilayer system. All the boundary conditions for solving local tasks in each liquid layer are presented in explicit form. The structure of the model allows it to expand by adding to it new data on the physical processes ensured the implementation of conservation laws.

The idea of constructing a model is based on the decomposition of complex problem into unitary elements (modules) with some set of rules allowing their linkage with each other [4]. Each module represents a monotypic local model, within which the calculation is carried out independently of the other modules of the system, while the “set of rules” defining, for example, the common boundaries between the modules and the corresponding functional relations, allow connecting these modules back to the original system. In this case the decisive role is played by physical parameters and thermophysical properties of liquid layers, which control the convective flows in the layers and determine the position and shape of interfaces.

E-mail address: ovcharova@hydro.nsc.ru

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.06.123
0017-9310/

Nomenclature

t	time
x,y	Cartesian coordinates
v_s, v_n	tangent and normal velocities of points lying on interface
D	stress tensor
P	pressure
$\vec{s}$, $\vec{n}$	tangent and normal vectors
d	width of thermal beam
h_i/h_0	dimensionless thickness of layers
h_0	length scale
v_0	velocity scale
p_0	pressure scale
g_0	gravity scale
Dimensionless parameters	
Pr_i	Prandtl number in i-th layer
Ca_i	Capillary number
Mn_i	Marangoni number
Cr_i	Crispation number
G	Galileo number
Greek symbols	
ψ	stream function
ω	vorticity
θ	temperature
ν	kinematic viscosity
ρ	density

2. Mathematical model

Consider a multilayer system consisting of thermally-conductive immiscible liquid layers as shown in Fig. 1. Here G_i are liquid layers, $f_j\,(t,x)$ are the interfaces between them. The bottom layer is lying on a solid substrate $f_J(t,x)$, the top layer is in contact with the air through the interface $f_0\,(t,x)$; at the left and right the system of layers is bounded by two solid planes $x = 0$ and $x = L$. Each liquid layer G_i is characterized by its thickness h_i, density ρ_i, kinematic viscosity ν_i and surface tension coefficient $\sigma_i(T)$. Moreover, the evaporation and condensation processes are not taken into account in this model; ρ_i and ν_i are considered to be constant.

Such a complex system of immiscible liquid layers can be represented as a flat ribbon graph consisting of two different types of elements: liquid layers G_i, ($i = 1,\ldots,I$) and the interfaces between them f_j ($j = 1,\ldots,J-1$), which we will call the internal interfaces. For the unequivocal description of the graph we must determine the order of the conformity of its elements. With this purpose, we enumerate all the interfaces and all liquid layers, for example, as is done in Fig. 1. Selected numbering scheme must be fulfilled during the whole process of solving the problem. According to the chosen numbering scheme, just one liquid layer with the number $i+1$ adjoins to internal interface from the bottom, while the layer with the number i adjoins from the top of the interface. Each liquid layer G_i is limited from above by the interface $f_j\,(t,x)$, while from below – by the interface $f_{j+1}\,(t,x)$. Surfaces $x = 0$, $x = L$, $f_0\,(t,x)$ and $f_J\,(t,x)$ have certain features and are classified as a special type surfaces. The orientation of the graph in the plane is presented in Fig. 1.

2.1. Consider the first type of graph elements – liquid layers

The fluid flow and heat transfer in each of the liquid layer G_i, ($i = 1,\ldots,I$) is described by the system of Navier-Stokes and thermal conductivity equations, which in the variables ψ, ω, θ (stream function, vorticity and temperature) have the form [5,6]

$$\frac{\partial\omega_i}{\partial t}+\frac{\partial}{\partial x}\left(\omega_i\frac{\partial\psi_i}{\partial y}\right)-\frac{\partial}{\partial y}\left(\omega_i\frac{\partial\psi_i}{\partial x}\right)=\frac{1}{Re_i}\Delta\omega_i, \tag{1}$$

$$\Delta\psi_i+\omega_i=0, \tag{2}$$

$$\frac{\partial\theta_i}{\partial t}+\frac{\partial}{\partial x}\left(\theta_i\frac{\partial\psi_i}{\partial y}\right)-\frac{\partial}{\partial y}\left(\theta_i\frac{\partial\psi_i}{\partial x}\right)=\frac{1}{Re_iPr_i}\Delta\theta_i \tag{3}$$

$$u_i=\frac{\partial\psi_i}{\partial y},\quad v_i=-\frac{\partial\psi_i}{\partial x},\quad \omega_i=\frac{\partial v_i}{\partial x}-\frac{\partial u_i}{\partial y}.$$

The characteristic values (scale) of length x_0, velocity v_0 and pressure p_0 should be the same for each layer of the system of layers. Here $Re_i = v_0\,h_i/\nu_i$ and $Pr_i = \nu_i/\chi_i$, where χ_i is the thermal diffusivity coefficient, are the Reynolds and the Prandtl numbers in i-th layer of the system, respectively. $\theta_i=(T_i-T_0)/\delta T$, where T_0 is the characteristic value of the temperature; δT is the characteristic temperature difference for the entire system of layers.

In addition, each liquid layer is characterized by its surface tension coefficient $\sigma_i(T)$ at the interface between the liquid and air. We assume that the surface tension coefficient of each liquid is a linear function of temperature

$$\sigma_i(T)=\sigma_0^{(i)}(1-\sigma_T^{(i)}(T_i-T_0)),\quad \sigma_0^{(i)}=\sigma_i(T_0),\sigma_T^{(i)}=-\frac{1}{\sigma_0^{(i)}}\frac{d\sigma_i}{dT}\bigg|_{T=T_0},\sigma_T^{(i)}>0.$$

i.e. it decreases linearly with the increase of temperature.

2.2. Interfaces. functional relations

Interfaces are the second type of graph elements. Consider any internal interface between the two layers G_i and G_{i+1}. As we consider any internal surface, we can define it by the equation $y = f\,(t,x)$. It is however important to keep in mind that the liquid with number $i+1$ lies below this interface, while the liquid with the number i is over it (see Fig. 2). It is related to the fact that when constructing the model we predetermined the order of conformity of graph elements.

Define the unit tangent and normal vectors at the interface $f\,(t, x)$ of liquids as

$$\vec{s}=\left\{\frac{1}{\sqrt{1+f_x^2}},\frac{f_x}{\sqrt{1+f_x^2}}\right\},\quad \vec{n}=\left\{\frac{-f_x}{\sqrt{1+f_x^2}},\frac{1}{\sqrt{1+f_x^2}}\right\}$$

Note that for liquid G_{i+1}, $\vec{n}$ is the outer normal unit vector at the interface of liquids, while for the liquid G_i for the same interface this normal unit vector is directed into the liquid.

2.2.1. Functional relations for temperature

In this paper we do not consider the evaporation (condensation) process at the liquid interface and neglect energy consumption related to its deformation (since they are negligible). Therefore,