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Arbitrarily polygonal transient thermal cloaks with natural bulk materials in bilayer configurations

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ABSTRACT

Transient thermal cloak has been attracting considerable attention due to the excellent performance in terms of heat-front maintenance and stable heat protection. However, their inhomogeneous and extreme parameters usually require challenging realization with metamaterials, thus leading to time-consuming and complicated fabrication even after some simplifications. In this paper, we propose a novel method to design an arbitrarily N-sided polygonal transient thermal cloak with nonsingular and homogenous material parameters. We further demonstrate that such cloak can be realized with only four kinds of natural bulk materials in bilayer configuration throughout. The multilayered materials can be easily arranged in a planarly stratified configuration due to all the flat boundaries, which makes the proposed cloaks much easier in fabrication and application. Simulation results validate the proposed scheme applicable to both steady-state and time-dependent cases with excellent performance. Based on the fact that an arbitrary shape can be approached by a polygon, our work actually opens up an avenue to arbitrarily shaped transient thermal cloaks made with natural bulk materials in bilayer configurations.

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1. Introduction

In the past few years, many significant achievements of invisibility cloaking have been motivated thanks to the pioneering works of Pendry [1] and Leonhardt [2]. By mapping the coordinate transformation to a set of permittivities and permeabilities, electromagnetic (EM) waves can be excluded from the cloaking region without perturbing the exterior fields [3,4]. Decoupling electric and magnetic effects, the static electric cloak [5,6] and the static magnetic cloak [7] have been also experimentally demonstrated. In addition to manipulation of EM/dc fields [3–7], the theoretical tool of coordinate transformation has been extended to acoustic cloaking [8,9], matter waves cloaking [10,11], and elastic waves cloaking [12,13]. Such an extension is based on the fact that the governing equations describing these systems are form-invariant under coordinate transformations [14]. Beyond the scope of the systems exhibiting waves, the theoretical tool of coordinate transformation is also valid for the manipulation of thermal conduction [15–17].

On the basis of the form-invariance of the thermal conduction equation, conventional circular thermal cloak was first investigated [15–17], following which arbitrarily shaped cloaks were also reported [18,19]. However, the material parameters are not only singular but also inhomogeneous and anisotropic, which is attributed to the transformation that is performed by expanding a point into a region along the radial direction [15–19]. To overcome the bottleneck, more feasible strategy has been proposed for the design of the thermal cloak using multilayered structure (where only anisotropic conductivities are needed), as demonstrated both experimentally [20] and theoretically [21]. To further simplify the implementation, the bilayer thermal cloaking has been experimentally demonstrated in two dimensions [22] and three dimensions [23]. The foregoing investigations are very attractive, however, all aforementioned thermal cloaking is strictly established in the steady state [15–24]. The steady-state thermal conduction obeys the Laplace equation, while the transient thermal conduction obeys diffusion equation [25]. Through tailoring the inhomogeneity and anisotropy of conductivities (as well as the specific heat and material density), transient thermal cloaking has been realized with metamaterials [26–28]. More recently, a diamond-shaped transient thermal cloak was investigated by expanding a line element rather than a point into a region, thus resulting in nonsingular and homogenous material parameters [29]. In addition, entropy analysis has been introduced to characterize the thermodynamics properties of two-dimensional and three-dimensional cloaks consisting of multiple layers [30–32].

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In this paper, we demonstrate a novel design of arbitrarily N-sided polygonal transient thermal cloaks with nonsingular, homogenous, and isotropic material parameters. The contribution of this study is threefold. First, the proposed transient cloak is inherently homogeneous in a Cartesian coordinate compared with the previous cloak with inhomogeneous materials [25–28]. Second, such cloaks can be realized with only four kinds of natural bulk materials arranged in a planarly stratified configuration, which is much more advanced compared to the cylindrical cloak that must be implemented by using $2N$ types of different isotropic materials, where N is the total number of discretization layers [25,26]. Third, the method proposed in this paper actually opens up an avenue to arbitrarily shaped transient thermal cloaks made with nonsingular and homogenous materials. We emphasize that the diamond-shaped transient thermal cloak [29] is a special case of our established general design road map.

2. Theoretical analysis

Heat flows spontaneously from a high temperature region toward a low temperature region. For a transient state and without a heat source, the general thermal conduction equation in the original space (x, y, z) can be written as

$$\nabla \cdot (\kappa \nabla T) - \rho c \frac{\partial T}{\partial t} = 0 \tag{1}$$

where T represents the temperature. Moreover, κ, ρ, and c are the thermal conductivity, the density, and the specific heat capacity of the background material, respectively. According to the transformation thermodynamics proposed by Guenneau et al. [25], it is known that the thermal conduction equation is form-invariant under coordinate transformation. Therefore, the thermal conduction equation in the transformed space (x', y', z') can be written as

$$\nabla' \cdot (\overleftrightarrow{\kappa}' \cdot \nabla' T') - \rho' c' \frac{\partial T'}{\partial t} = 0 \tag{2}$$

Note that the material parameters in the transformed space can be expressed as

$$\overleftrightarrow{\kappa}' = \frac{\mathbf{A} \kappa \mathbf{A}^{\mathrm{T}}}{\det(\mathbf{A})}, \quad \rho' c' = \frac{\rho c}{\det(\mathbf{A})} \tag{3}$$

where $\mathbf{A} = \frac{\partial(x', y', z')}{\partial(x, y, z)}$ is the Jacobian transformation matrix.

For simplicity but without loss of generality, we consider here a two-dimensional case. Fig. 1 shows the schematic of the coordinate transformation for the design of an arbitrarily N-sided polygonal thermal cloak. In Fig. 1(a), an arbitrarily N-sided polygonal is used in the original space, and in its center is a smaller N-sided polygon rotated at an angle of π/N compared to the outside polygon. The radii of the center polygon and the outer polygon are r_0 and r_2, respectively. We divide the space between the two polygons into several triangular elements. It is clear that these triangular elements can be grouped into two types, which we marked as Element I (light yellow) and Element II (light blue) with their local coordinate axes $(u_{\mathrm{I}}, \upsilon_{\mathrm{I}})$ and $(u_{\mathrm{II}}, \upsilon_{\mathrm{II}})$, respectively. A linear compression transformation along the local coordinate axes is applied in all elements, so that the center polygon with radius r_0 in the original space (Fig. 1(a)) is transformed to a bigger polygon with radius r_1 in the transformed space (Fig. 1(b)), while the outer polygon with radius r_2 remains unchanged in the transformation. Thus an arbitrarily N-sided polygonal thermal cloak is formed as shown in Fig. 1(b), where the region enclosed by the polygon with radius r_1 is used to hide the target. It is noted that the smaller is the value of r_0 (which means that the polygon with radius r_0 is closer to a point), the better is the cloaking performance.

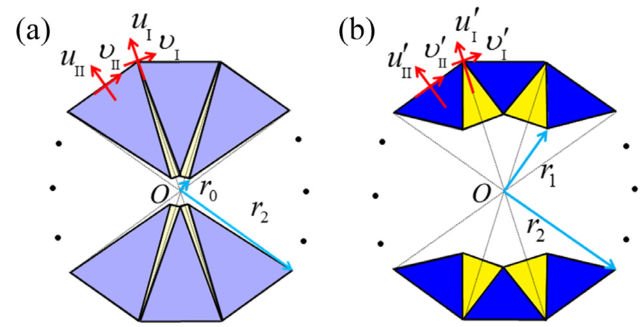

Fig. 1. Schematic of coordinate transformation for the design of an arbitrarily N-sided polygonal thermal cloak. (a) Original space. (b) Transformed space.

Following the analysis outlined above, the transformation equations for Element I can be expressed as

$$u'_{\mathrm{I}} = \frac{r_2 - r_1 \cos(\pi/N)}{r_2 - r_0 \cos(\pi/N)} u_{\mathrm{I}}, \quad \upsilon'_{\mathrm{I}} = \frac{r_1}{r_0} \upsilon_{\mathrm{I}} \tag{4}$$

Submitting Eq. (4) into Eq. (3), the material parameters for Element I (yellow[1] triangles of Fig. 1(b)) can be obtained as

$$\kappa'_{u_{\mathrm{I}}} = \frac{(r_1 \cos(\pi/N) - r_2) r_0}{(r_0 \cos(\pi/N) - r_2) r_1} \kappa, \quad \kappa'_{\upsilon_{\mathrm{I}}} = \frac{(r_0 \cos(\pi/N) - r_2) r_1}{(r_1 \cos(\pi/N) - r_2) r_0} \kappa \tag{5a}$$

$$\rho'_{\mathrm{I}} c'_{\mathrm{I}} = \frac{(r_0 \cos(\pi/N) - r_2) r_0}{(r_1 \cos(\pi/N) - r_2) r_1} \rho c \tag{5b}$$

Similarly, the transformation equations for Element II can be expressed as

$$u'_{\mathrm{II}} = \frac{r_1 - r_2 \cos(\pi/N)}{r_0 - r_2 \cos(\pi/N)} u_{\mathrm{II}}, \quad \upsilon'_{\mathrm{II}} = \upsilon_{\mathrm{II}} \tag{6}$$

Submitting Eq. (6) into Eq. (3), the material parameters for Element II (blue triangles of Fig. 1(b)) can be obtained as

$$\kappa'_{u_{\mathrm{II}}} = \frac{r_2 \cos(\pi/N) - r_1}{r_2 \cos(\pi/N) - r_0} \kappa, \quad \kappa'_{\upsilon_{\mathrm{II}}} = \frac{r_2 \cos(\pi/N) - r_0}{r_2 \cos(\pi/N) - r_1} \kappa \tag{7a}$$

$$\rho'_{\mathrm{II}} c'_{\mathrm{II}} = \frac{r_2 \cos(\pi/N) - r_0}{r_2 \cos(\pi/N) - r_1} \rho c \tag{7b}$$

The N-sided polygonal cloak in Fig. 1(b) is formed by lots of such elements, whose materials are homogeneous and anisotropic, as illustrated in Eqs. (5) and (7). In addition, no singular values of the material parameters are involved of the N-sided polygonal cloak as long as $r_0 > 0$. This is an important improvement compared with the cylindrical thermal cloak proposed by previous work [25]. Due to the proposed cloak with finite constant parameters, it could be easily realized through alternating layered isotropic medium and only two types of isotropic materials (medium A and medium B) are needed for each element. Assuming that the cloak consists of two homogeneous isotropic layers with thickness of d_A and d_B, conductivities κ_A and κ_B, densities ρ_A and ρ_B, heat capacities c_A and c_B, respectively, we obtain the effective parameters based on effective medium theory (EMT) as following [25]

$$\frac{1}{\kappa'_{uii}} = \frac{1}{1+\eta} \left(\frac{1}{\kappa'_{Ai}} + \frac{\eta}{\kappa'_{Bi}} \right), \quad \kappa'_{vii} = \frac{\kappa'_{Ai} + \eta \kappa'_{Bi}}{1+\eta} \quad (i = \mathrm{I}, \mathrm{II}) \tag{8}$$

$$\rho'_i c'_{ii} = \frac{\rho_{Ai} c_{Aii} + \eta \rho_{Bi} c_{Bi}}{1+\eta} \quad (i = \mathrm{I}, \mathrm{II}) \tag{9}$$

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

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