International Journal of Heat and Mass Transfer 115 (2017) 50–61

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Mean equation based scaling analysis of fully-developed turbulent channel flow with uniform heat generation

Ang Zhou [a,*], Sergio Pirozzoli [b], Joseph Klewicki [a,c]

[a] Department of Mechanical Engineering, University of New Hampshire, Durham, NH 03824, USA
[b] Dipartimento di Ingegneria Meccanica e Aerospaziale, Università di Roma 'La Sapienza', Via Eudossiana 18, 00184 Roma, Italy
[c] Department of Mechanical Engineering, University of Melbourne, Victoria 3010, Australia

ARTICLE INFO

Article history:
Received 10 May 2017
Received in revised form 2 August 2017
Accepted 4 August 2017

Keywords:
Wall-turbulence theory
Scalar and heat transport
Re and *Pr* scaling

ABSTRACT

Multi-scale analysis of the mean equation for passive scalar transport is used to investigate the asymptotic scaling structure of fully developed turbulent channel flow subjected to uniform heat generation. Unlike previous studies of channel flow heat transport with fixed surface temperature or constant inward surface flux, the present flow has a constant outward wall flux that accommodates for the volumetrically uniform heat generation. This configuration has distinct analytical advantages relative to precisely elucidating the underlying self-similar structure admitted by the mean transport equation. The present analyses are advanced using direct numerical simulations (Pirozzoli et al., 2016) that cover friction Reynolds numbers up to $\delta^+ = 4088$ and Prandtl numbers ranging from $Pr = 0.2$–1.0. The leading balances of terms in the mean equation are determined empirically and then analytically described. Consistent with its asymptotic universality, the logarithmic mean temperature profile is shown analytically to arise as a similarity solution to the mean scalar equation, with this solution emerging (as $\delta^+ \to \infty$) on an interior domain where molecular diffusion effects are negligible. In addition to clarifying the Reynolds and Prandtl number influences on the von Kármán constant for temperature, k_θ, the present theory also provides a couple of self-consistent ways to estimate, k_θ. As with previous empirical observations, the present analytical predictions for k_θ indicate values that are larger than found for the mean velocity von Kármán constant. The potential origin of this is briefly discussed.

1. Introduction

Wall-bounded turbulent flows pervade industrial applications. This fact broadly motivates the on-going efforts to investigate the properties of wall-flows. In this regard, the associated transport of heat, mass and momentum are of particular technological importance in applications pertaining to energy efficiency, environmental concerns, and manufacturing processes.

Broadly speaking, thermal processing seeks to force a temperature variation in a system to attain a specific goal, while the purpose of thermal control seeks to regulate the temperature within specified bounds, or to control the temperature over time within a certain margin to ensure a desired operating condition. For such aims, prediction across parameter variations is often an important consideration. Thus, significant efforts have been devoted toward quantifying scaling behaviors associated with heat transport. Analyses involving the application of multiple-scale approaches are often used to explore parameter dependent properties of statistical profiles, e.g. [1–4]. An especially prominent scaling framework is based upon the notion of an *overlap layer*, as adopted from the mathematical machinery of matched asymptotic expansions. Here it is postulated that there exists a region where respective functions of inner and outer normalized distance from the wall are simultaneously valid [5,6]. An alternative approach, that more directly invokes the idea of *distance-from-the-wall* scaling, can be deduced from dimensional analysis [7]. Under this assumption, Townsend's attached eddy phenomenology is inherently consistent with the existence of a logarithmic mean velocity profile [8–10]. In connection with this, more recent studies reveal that the streamwise velocity variance, as well as their higher order even moments also vary logarithmically [11–13] in a region that falls within the bounds of the mean profile logarithmic layer. Within this region (inertial sublayer), the mean dynamics are dominated by inertia, and the mean momentum equation admits a self-similar structure [14].

* Corresponding author.
E-mail addresses: zhouang48@gmail.com (A. Zhou), sergio.pirozzoli@uniroma1.it (S. Pirozzoli), joe.klewicki@unh.edu, klewicki@unimelb.edu.au (J. Klewicki).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.08.009
0017-9310/

Consistent with the analogy between heat and momentum transport, it is rational to expect that the mean temperature profile in a heated wall-flow will exhibit properties similar to those of the mean velocity profile. Kader [15] describes the law-of-the-wall for temperature in a manner similar to the inner function in the overlap framework for velocity. This formulation follows from the assumption that, in the near wall region, the mean temperature, Θ, depends only on the shear stress at the wall, τ_w, the heat flux at the wall, q_w, the distance from the wall, the mass density, ρ, dynamic viscosity, μ, specific heat, C_p, and thermal conductivity, k. Accordingly, a logarithmic profile for temperature is observed for inner-normalized distances from the wall greater than about 30 and Prandtl number less than 1, e.g., [16]. As such, the overlap layer approach has been used to reason the logarithmic structure of the thermal boundary layer [17,18]. Based on the overall mean temperature profile structure, the inner region close to the solid wall is seen to be composed of a molecular sublayer and a thermal buffer layer, while logarithmic and wake layers comprise an outer region that extends to the centerline of the channel/pipe. Like for velocity, some divide the logarithmic (overlap) layer into two sublayers [19,20]. In this description, a convective sublayer is characterized by a negligible conductive effect, while heat transfer is under a detectable influence of conduction in the thermal mesolayer.

Based upon his review of available data, Kader [18] estimated that the thermal Kármán constant, k_θ, in the logarithmic mean temperature profile equation is about 0.47. For a fully developed turbulent channel flow with uniform heating from both walls k_θ was found by Kawamura et al. [21] to be roughly independent of Reynolds number and close to the Kármán constant for velocity, i.e., $0.40 \lesssim k_\theta \lesssim 0.42$. It is relevant to note, however, that the law-of-the-wall for temperature apparently breaks down in flows where retains validity for velocity. Here we note that the logarithmic increase in mean temperature has been previously purported to be more sensitive to a streamwise pressure gradient than the mean velocity [22].

Interest in statistical profile properties in wall turbulent flows has motivated approaches that more directly incorporate the mean equations to discern scaling behaviors. Based on the relative magnitude of terms in the mean momentum equation, Wei et al. [23] revealed a four layer structure distinct from the traditional description. As expected, layers I and IV (the innermost and outermost layers) respectively comply with inner and outer scaling. However, an intermediate length scale, $\sqrt{\nu\delta/u_\tau}$, is both empirically observed and analytically shown to characterize the other two layers. Similarly, Afzal and coworkers [24–26] deduced an intermediate scaling for the thermal meso-layer of fully-developed turbulent channel flow and transitionally rough channel flow. Their analysis incorporates an intermediate layer that has its own characteristic scaling, and that lies between the traditional inner and outer layers. Their formulation also employs a matching procedure that incorporates three layers and two overlapping regions over which two adjacent logarithmic regions for the mean temperature profile are shown to asymptotically form. The thermal meso-length scale they employ constitutes the geometric mean of the inner, α/u_τ, and outer, δ, thermal length scales, and under inner normalization is given by $\sqrt{Pr\delta^+}$. Here, α is the thermal diffusivity, δ is the half channel height, u_τ is the friction velocity and Pr is the Prandtl number. Afzal's analysis similarly employs an intermediate scaled temperature $T_m = (\Theta_w + \Theta_c)/2$, where Θ_w and Θ_c are the temperature at the wall and the channel centerline, respectively.

Using an analysis that also incorporates an intermediate scale, Wei et al. [27] examined fully developed thermal transport in channels with constant wall heat flux. They introduced a new inner variable, $y_\sigma = \eta/\sigma^2$, where $\eta = y/\delta$ and σ is a parameter defined as a function of δ^+ and Peclet number, $Pe_\tau = Pr\delta^+$. Consequently, the corresponding thermal mesoscale, $\sqrt{(\Theta_w - \Theta_c)/(\Theta_\tau Pr\delta^+)}$ where $\Theta_\tau = q_w/\rho C_p u_\tau$, is different from the geometric mean of the inner and outer thermal length scales, $\sqrt{\alpha\delta/u_\tau}$. Existing DNS, however, significantly limited the range of parameters over which Wei et al. could validate their analysis. Based on DNS data covering a range of both Reynolds and Prandtl numbers, Saha et al. [28] explored the scaling properties of scalar transport under a larger range of constant wall flux conditions. Based upon the magnitude ordering of terms, they showed that the four-layer thermal regime emerges when $Pr \gtrsim 0.6$ at $\delta^+ = 180$. This four layer regime is analogous to that first identified by Wei et al. [23] for the momentum, and its onset occurs at a similar δ^+ [29]. The analysis of Saha et al. incorporates the inner normalized mesoscale, $\sqrt{Pe_\tau}$, which they show can be used to effectively merge both the mean temperature and turbulent heat flux over a domain that starts interior to the location of peak heat flux and ends near the channel centerline.

The previous analyses of the mean thermal energy equation by Wei et al. [27] and Saha et al. [28] investigated flows having a constant surface heat flux boundary condition. Analytically, this presents a significant challenge when compared to the corresponding streamwise momentum equation analysis, where the pressure gradient in the inner-normalized form of the equation is represented by $1/\delta^+$. Additionally, the low Reynolds numbers of previous data make it difficult (and less convincing) to validate the veracity of the analytical results associated with an asymptotic analysis. In particular, their data analyses of the mean scalar equation failed to provide comparably compelling evidence for the existence of a scaling layer hierarchy (i.e., analogous to what has been shown for the mean momentum equation), nor clearly delineate trends for varying Reynolds number and Prandtl number.

The present study follows the same methodology used in previous studies of the mean momentum [23] and kinetic energy budgets [30]. Here, however, we investigate the mean scalar balance equation with uniform heat generation for the fully-developed turbulent channel flow. The uniform heat generation term addresses (analytically mitigates) previous challenges just mentioned by reducing the mean scalar equation into a form that is much more like the mean momentum equation. Herein we also employ DNS data covering a significantly larger range of Reynolds and Prandtl numbers. The net result of this is to provide more compelling support for the analytical findings, and more clearly expose δ^+ and Pr trends.

In what follows, the ratio of the molecular diffusion (MD) term to the gradient of turbulent heat flux (GT) term in the mean scalar equation (e.g., Eq. (7) below) is employed to reveal a four-layer leading balance structure. Both the Reynolds number and Prandtl number dependent properties of these layer thicknesses is then empirically quantified with DNS data and verified through analysis of the mean equation. As with the momentum field, the analysis also indicates that the mean scalar equation can be cast into an invariant form that properly reflects the local dominant physical mechanisms, and which exposes the effect of the governing small parameter on an intrinsic scaling layer hierarchy. The Prandtl number impact on the width distribution of the layer hierarchy is quantified and discussed relative to the underlying physics. Consistent with the theory, it is shown that on the scaling layer hierarchy there exists a domain where molecular diffusion effects are subdominant. Here, the layer width function becomes proportional to the distance from the wall. On this domain, the mean equation is shown to asymptotically admit a similarity solution in the form of a logarithmic mean temperature profile. The behaviors of the coefficients in the logarithmic expression, including k_θ, are also described.