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Finite element modeling of the Hot Disc method

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ABSTRACT

The Hot Disc method, also known as the transient plane source (TPS) technique, is an experimental approach to determine the thermal transport properties of materials. The core of the method is the Hot Disc sensor, an electrically conducting metallic strip, shaped as a double spiral clad with a protective polymer film. The mean temperature increase in the sensor has been approximated from various analytical approaches such as: the concentric ring sources model, the thermal quadrupoles formalism, and concentric circular strips structure approach. However, full numerical simulation of the sensor has not been addressed so far. Here we develop a 3D model of Hot Disc sensors and compare simulated mean temperature increase to experimental recordings. Joule heating coupled with heat transfer of solids (of COMSOL Multiphysics software) is used to simulate the working principle of the sensor. The volume mean temperature increase in the sensor from the simulations proves to be in a good agreement with the corresponding experimental recordings. The temperature distributions of the metallic strip are also evaluated and discussed with respect to the previous experimental findings. Furthermore, the current distribution across the strip is obtained. Such simulation can potentially be used in further optimizing geometry and parameter estimation.

1. Introduction

The thermo-physical properties of materials such as thermal conductivity, thermal diffusivity, and specific heat capacity have been characterized using different methods developed over the years [1,2,6,7,11,12]. Measuring the thermal conductivity of materials is crucial to a wide range of applications including insulation for space rockets, home insulation using various building materials, thermal management in electronics and gas turbines, and so on. In addition, the knowledge of thermal conductivity of materials can be a useful input in processes involving heat and mass transfer such as the rotating disk problems [18,21] and heat transfer in living tissues [22].

The Hot Disc method, the focus of this study, is a primary experimental tool for thermal property measurement of materials. It is an extension of the hot strip method [8] which is designed to cover a large range of materials with a wide variation in thermal transport properties. The method uses a single object (Hot Disc sensor) as a heating and sensing unit. This unit is made from a nickel foil arranged in a spiral fashion. The heat of the spiral is generated through resistive heating when current is allowed to pass through it.

The heat conduction equation for an isotropic material with thermal diffusivity κ, volumetric heat capacity ρc and heat source Q is given by [5]

$$\kappa\nabla^2 T(\vec{r},t) + \frac{Q(\vec{r},t)}{\rho c} = \frac{\partial T(\vec{r},t)}{\partial t}. \quad (1)$$

where ρ is density, c is specific heat of the material, $T(\vec{r},t)$ is temperature and time t. The fundamental solution $(Q = 0, t > 0)$ for Eq. (1) is given by [10]

$$T = T_o + \frac{1}{(4\pi\kappa t)^{\frac{3}{2}}} exp\left(-\frac{r^2}{4\kappa t}\right). \quad (2)$$

where T_o is the initial temperature. However the general solution with heating source of $Q(\vec{r},t)$ is given by the convolution of the function $\frac{Q(\vec{r},t)}{\rho c}$ with the above fundamental solution (Eq. (2)), which requires integration over the source volume. For the Hot Disc sensor, the double spiral source volume can be approximated by concentric ring sources of heating power $P_o = \pi a(m+1)Q_o$, where Q_o is heat released per unit length, m is the number of concentric rings, a is the radius of the largest ring and the total heating length is

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expressed as $\sum_{l=1}^{m} 2\pi la/m = (m+1)\pi a$. Using dimensionless integration parameter $\sigma^2 = \frac{\kappa(t-t')}{a^2}$, (for $t'=0$ the characteristic time (τ) is expressed as $\tau = \frac{\sqrt{\kappa t}}{a}$) and thermal conductivity of the material $K = \kappa\rho c$, the average change in temperature on the Hot Disc sensor is given by [10]

$$\overline{\Delta T}(\tau) = \frac{1}{(m+1)\pi a} \frac{P_o}{2\pi^{\frac{3}{2}} am(m+1)K} \int_0^{\tau} \frac{d\sigma}{\sigma^2} \sum_{k=1}^{m} \frac{ka}{m} \times \sum_{k=1}^{m} l exp\left\{-\frac{(\frac{k^2}{m^2}) + (\frac{l^2}{m^2})}{4\sigma^2}\right\} I_o\left(\frac{kl}{2m^2\sigma^2}\right) 2\pi \tag{3}$$

$$\overline{\Delta T}(\tau) = A + \frac{P_o}{\pi^{\frac{3}{2}} aK} D(\tau) \tag{4}$$

A is a constant which represents the initial temperature increase, P_o is power input, and I_0 is modified Bessel function. The dimensionless shape function $D(\tau)$ is given by [6,10].

$$D(\tau) = \frac{1}{m^2(m+1)^2} \int_0^{\tau} \frac{d\sigma}{\sigma^2} \sum_{k=1}^{m} k \sum_{l=1}^{m} l exp\left\{-\frac{(\frac{k^2+l^2}{m^2})}{4\sigma^2}\right\} I_o\left(\frac{kl}{2m^2\sigma^2}\right) \tag{5}$$

The experimental measurement provides the time dependent resistance of the TPS element using the expression:

$$R(t) = R_o[1 + \alpha\overline{\Delta T}(\tau)] \tag{6}$$

Here, R_o is the resistance of the TPS element before the transient recording and α is the temperature coefficient of the resistance. Finally, using Eq. (6) to determine the temperature development on the sensor and fitting it to Eq. (4), one can extract the thermal properties of the material.

The complexity of the shape function leads to different assumptions and approaches to get approximate analytical or numerical solutions.

He [10] discussed the theory of Hot Disc method from the first principle approach. Solving the heat conduction equation of an isotropic material starting from its point source solution and extending it to a single and multiple concentric line source solutions, as discussed in Eqs. (1)–(6). In his detailed report the author asserts that the concentric ring models could provide a similar expression as in the original TPS method [6] and with proper correction, the Hot Disc technique could provide an accurate measurement of thermal conductivity and diffusivity of a wide variety of materials.

The concentric circular strip model [14] is aimed at improving the representation of the original model to the actual sensor. It considers a set of concentric strips of finite width. The temperature shape function is approximated as a 'circle' model and a 'ring' model. The 'circle' model was shown to have solved the issue of infinite limit of the shape function for small τ values, by introducing the modified shape function.

In the thermal quadrupolar complete approach for the Hot Disc method [13]: the author questions the original Hot Disc method assumptions [6] such as effect of sensor inertia and the assumption of the heat source as concentric ring with negligible width, by pointing to the singularity that exit when τ goes to zero. The report proposes an alternative model which considers both the thermal contact resistance and heat capacity of the sensor using thermal quadrupolar formalism.

In the original Hot Disc method [6], two approximations for experimental arrangements were proposed; the Hot Square and the Hot Disc. In this work, we consider the Hot Disc approximation. The experimental measurements of the Hot Disc method provide the average rise in temperature across the entire sensor volume. Eq. (4) is fitted to these experimentally recorded values from which the thermal properties of the surrounding material can be extracted. A time correction, t_c, is included to address the non-zero rise of the heating power due to the heat capacity of the sensor itself. The time correction and thermal diffusivity is iterated until a best fit is obtained. The nickel strip is embedded into a polymer film for electrical insulation and mechanical support. The effect of this film and thermal contact resistances are also taken into account in the TPS method [11]. Different applications of the Hot Disc method are reported in literature [4,6,9,15,17,20].

The different approaches consider a form of circular concentric heat sources with negligible heat capacity placed on an infinite medium. In the present paper, the simulation analyses the actual double spiral of the sensor placed on a sample with limited dimensions. Therefore, the simulation can capture the actual spiral geometry and include all dimensions and properties of both the sensor and the sample. In addition, the simulation can provide the details on the temperature and current distribution on the sensor, which are hard to measure in an experiment. Recently, a 2D numerical study on the Hot Disc method was reported [24], it addresses the effect of the covering layer of the Hot Disc sensor. The aim of this work is to develop a full 3D model, where the temperature and current profiles of the sample and the sensor are derived.

The remainder of this paper is organized as follows. In Section 2, the numerical and experimental methods are described. Section 3 contains numerical and experimental results with their discussion and comparison. Conclusions are made in Section 4. Details of the simulation parameters are listed in Appendix. A supplementary file contains the simulation film that demonstrates the propagation of heat in the Hot Disc method.

2. Method

The working principle of the Hot Disc method is simulated by considering a double spiral nickel strip embedded in a Kapton layer, which in turn is sandwiched between two identical polymethyl methacrylate (PMMA) samples. A similar experimental procedure is employed for comparison.

2.1. Numerical simulation

COMSOL Multiphysics software is used to compute the numerical solution of the temperature distribution in the system. The software uses the finite element method. The standard simulation procedure in COMSOL includes choosing the physics and solver type, defining geometry and materials, applying appropriate boundary conditions and identifying the meshes type which provides mesh independent results. Modeling the properties of the Hot Disc system (choosing the physics and solver type) is achieved by coupling the electric current heating source with heat transfer in solids module of COMSOL Multiphysics software. In the Hot Disc method, electric current supplied to the double spiral element generates heat by resistive heating. The temperature and thus resistance of the sensor element increase as a function of time (Eq. (6)), while current is supplied. The shape of the resulting transient curve (temperature vs time) depends on the thermal properties of the surrounding material. For example, the temperature rise is quicker for a sample with low thermal conductivity.

The modeling is done through a two-step simulation: static and transient. Both the simulations require the construction of double spiral geometry of the sensor (Fig. 1(right)). This is achieved by using the parametric equation,

$$x = \left(\frac{s}{c_1\pi}\right)\sin(s) \text{ and } y = \left(\frac{s}{c_2\pi}\right)\cos(s) \tag{7}$$

where s is the parameter that runs from 0 to $n\pi$ and n determines the number of rings. The constant c_1 and c_2 affects the radius increase of each ring of the sensor. The dimensions of the simulated