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Direct numerical simulation of natural convection in a square cavity with uniform heat fluxes at the vertical sides: Flow structure and transition

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A B S T R A C T

Direct numerical simulations have been performed of natural convection in a closed cavity with constant heat fluxes on the vertical sides. Results are presented for Rayleigh numbers in the range 3×10^{11}–4×10^{14} and Prandtl number equal to 1. The flow is characterized by two boundary layers near the heated and cooled vertical sides and two intrusion layers near the horizontal walls, which complete a recirculation pattern. The net energy surplus transferred upwards by both boundary layers is balanced by heat conduction through the main body of the cavity. The vertical boundary layer thickness is of $O(Ra^{-2/9})$ and the intrusion layers of $O(Ra^{-1/6})$. Scalings for thicknesses, velocities and temperature gradients, obtained from order of magnitude estimates, are confirmed by the numerical simulations. The bulk temperature gradient is much larger than those across the thin boundary layers and, therefore, effective in delaying the onset of instability. Unstable flows are thus found only above a Rayleigh number of the order of 10^{12}, in contrast to a cavity with fixed lateral wall temperatures where this is close to 10^8. The threshold for instability and the dominant frequencies are in satisfactory accord with relevant linear stability results from the literature. The bulk of the cavity responds passively and filters the excitation from the unstable boundary layers by retaining frequencies close to the Brunt-Vaisala frequencies in a density stratified fluid.

1. Introduction

Natural convection flows are encountered in a large number of natural phenomena and technological applications. Some well-known examples include flows in the atmosphere and the oceans, motion in the earth's mantle with the associated magnetic field and its reversals, recirculation in lakes, estuaries and ponds, heat exchange of organisms with their environment, fluid motion in storage and in heat transfer equipment, CVD and plasma processing, metrology, etc. [1–5]. Understanding and predicting such complex phenomena that exhibit chaotic behavior and a great multiplicity in length and time scales is very important for technological applications and even more so for natural phenomena, as for example in weather prediction, ocean currents and climatic change.

From a different perspective, such flows offer paradigms for transition to increasingly complex behavior and chaos. They comprise, therefore, an attractive area for theoretical studies, where numerical computation is, of course, an increasingly important tool. Many studies exist in the literature, which consider simplified and idealized geometries, i.e. fluids near heated flat plates, or contained within cylinders [6–8] and cavities [9–15]. It may be mentioned also that the complexity of phenomena together with the simplicity of some flow domains make such problems an attractive ground for testing the accuracy and efficiency of numerical techniques.

Several studies exist in the literature dealing with natural convection flow in a cavity, which is the topic of the present work as well. In particular, the case where the vertical sides are held at fixed temperatures includes several two- and three-dimensional computational studies [10–15] investigating flow structure and dynamics, transition, scaling regimes, heat transfer, dispersion of suspended microparticles [16], as well as efficiency of numerical approaches.

The related problem where the heat flux instead of the temperature is fixed on the lateral walls has received relatively less attention. The flow and heat transfer near a flat vertical plate has been considered in the works of Gebhart and collaborators [17–19] in terms of boundary layer structure and linear stability. Bark et al.

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Nomenclature

c_P	specific heat
$\boldsymbol{e_y}$	normalized unit vector in y-direction
f	frequency
f_{max}	maximum Brunt-Vaisala frequency
g	gravity constant
H	linear dimension of square cavity
k	thermal conductivity
k_x	wavenumber in the x-direction
k_y	wavenumber in the y-direction
$\boldsymbol{n}$	unit vector normal to the interface
P	normalized liquid pressure
P_0	pressure scale
Pr	Prandtl number
q	heat flux
r	dimensionless test function for pressure
Ra	Rayleigh number
Re	Reynolds number
S	vertical temperature gradient
t	time
T	temperature
T_0	reference temperature
$\boldsymbol{u}$	normalized velocity
U_0	velocity scale
U_I	velocity magnitude of intrusion layers
$\boldsymbol{v}$	dimensionless test function for velocity
x	spatial coordinate
y	spatial coordinate

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
δ	boundary layer thickness
δ_I	intrusion layer thickness
Δt	dimensionless numerical time step
ΔT	temperature difference across boundary layers
Θ	dimensionless temperature
μ	viscosity
ν	kinematic viscosity
ρ_0	density at reference temperature
Φ	dimensionless test function for temperature

Superscripts and subscripts

0	reference value
n	instantaneous value at n-th time step

[20] analyzed the scalings, the transient development of boundary layers, as well as the bulk stratification in a vertical slot with prescribed fluxes on the two opposing sides. Interestingly, they suggest that similar phenomena take place during mass transfer in electrochemical systems, i.e. in lead-acid batteries or and in electrochemical polishing of metals. The linear stability of the flow in a similar slot has been considered by Sundstrom and Vynnycky [21] for various orientations. McBain et al. [22] considered the flow near a flat plate in the presence of a mean temperature gradient in the bulk and presented results from linear stability analysis, as well as nonlinear simulations near the threshold of instability. Interestingly, they suggest that the fixed heat flux boundary condition describes more faithfully the processes during katabatic wind generation due to radiative cooling of the soil at night, a problem originally considered by Prandtl. Direct numerical simulations of the unstable boundary layer near a uniformly heated vertical plate but without a bulk temperature gradient are presented in Aberra et al. [23].

To the best of the authors' knowledge, despite its simplicity, the problem of flow development and transition to instability in a closed cavity with fixed heat fluxes on the sides has been only addressed to a very limited extend in the literature, and mostly for shallow cavities [24,25]. As will be discussed in the following sections, for cavities of aspect ratio near unity or larger (height over width), a challenge to be faced is that very high Rayleigh numbers are needed for instability to appear. This is because the heat flux conditions allow a stable temperature stratification to develop in the bulk, which overshadows the temperature differences in the boundary layers and suppresses the development of the instability.

The structure of the presentation is a follows: In Section 2 the problem is defined and the basic scalings for velocities, temperatures and boundary layer thicknesses are deduced on the basis of order of magnitude estimates for laminar flow. Then the dimensionless equations are obtained and the numerical approach is outlined. In Section 3 the results obtained from direct numerical simulations are presented over a range of Rayleigh numbers that covers the transition to instability and progressively more complex behavior. The basic scalings are compared with the numerical results and the features of the instability, such as the transition threshold and the frequencies of the unstable waves are discussed in the light of previous relevant studies.

2. Problem definition, mathematical and numerical formulation

We consider a square cavity enclosing a Boussinesq fluid. Heat is supplied uniformly through one vertical side and it is withdrawn from the opposite vertical side, also uniformly. Thus, the overall thermal balance is maintained in the cavity. The top and bottom boundaries of the cavity are considered to be thermally insulated. Owing to the development of temperature non-uniformities within the fluid, density differences arise resulting in natural convection in the cavity.

2.1. Mathematical description

The fluid motion is governed by the Navier-Stokes equations, augmented by the buoyancy term, which in the Boussinesq approximation is linear in the temperature deviation from a reference value, T_0,

$$\rho_0 \frac{D\boldsymbol{u}}{Dt} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho_0 g \beta (T - T_0) \boldsymbol{e}_y \tag{1}$$

Here, ρ_0 is the fluid density at the reference temperature, μ the viscosity and β the thermal compressibility, which are both assumed constant. This is supplemented by the continuity equation, which retains the same form as in incompressible flows.

$$\nabla . \boldsymbol{u} = 0 \tag{2}$$

No-slip conditions are assumed to apply over all the bounding surfaces of the cavity.

The temperature field is described by the energy equation

$$\rho_0 c_P \frac{DT}{Dt} = k \nabla^2 T, \tag{3}$$

where the thermal conductivity, k, and heat capacity, c_P, are also assumed constant. The constant heat flux conditions at the vertical boundaries take the form