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Improving the rate of heat transfer and material in the extended surface using multi-objective constructal optimization

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ABSTRACT

Constructal theory introduces a new method to optimize the various engineering applications such as, extended surfaces fins. The aim of this study is to design and analyze a cylinder in which its heat transfer surface area is increased by fins using the concept of constructal theory proposed by Bejan. Three configurations including three branches, two branches and simple fins are presented. By the increase in the rate of heat transfer the volume of material applied in the fin should be increased. Therefore, the optimization is done by considering the total heat transfer rate and fin material as two objective functions. In order to maximize the heat transfer rate as well as minimize the volume of material five parameters are selected as the design parameters. The effects of decision variables on the rate of heat transfer for the optimum points are presented. In addition, the effect of increase in the number of fin branches on the heat transfer rate is estimated. Also fin temperature profiles for the three studied configurations for two cases of heat transfer rates are represented to indicate the effects of theory. Finally, the volume of material applied in the fin for two types of copper and aluminum in the fixed value of heat transfer rate is obtained for the different configurations, and the results are discussed.

1. Introduction

Constructal theory can be considered as a new method for optimization of the different engineering applications such as, extended surfaces fins. In fact, constructal law is the law of physics that accounts for the phenomenon of evolution of organization throughout nature [1–6]. Heat transfer enhancement through finned surfaces is widely performed in many industrial applications [7]. Many research has been done in the recent years to seek for the optimal geometries of the fins by using constructal theory. Hazarika et al. and Kundu et al. determined the performance and optimization of a constructal T-shaped fin to predict the performance parameters and optimum design parameters [8–10]. Chen et al. and Bello-Ochende et al. studied the optimization of a cylindrical pin-fin based on constructal theory to find the effects of fin-material fraction and other parameters on optimization results and maximize the total heat transfer rate [11,12]. Gong et al. investigated the construct of a three-dimensional cylindrical heat sources with convection heat transfer by using constructal theory for the optimal design of a heat source system [13]. Lorenzini et al. applied the constructal design to perform the geometric optimization of the Y-shaped assembly of fins [14]. In other research, a constructal design was determined by Lorenzini et al. for the non-uniform X-shaped conductive pathways for cooling reason [15]. In another research, Lorenzini et al. applied the constructal concept to increase the heat flow through the X-shaped conductive pathways [16]. Thermal optimization of a fin shape was performed using Bejan's constructal theory in another research of Lorenzini et al. and Reis [17,18]. The optimal construct of T-shaped assembly was proposed and compared with the optimal construct with maximum temperature difference minimization by Chen et al. [19]. Chen et al. studied about a tree-shaped assembly of fins and the obtained results were compared with those of T-shaped assembly with Constructal entransy dissipation rate minimization [20]. The effects of adding horizontal fins on the efficiency of a phase change material (PCM) heat sink was studied based on constructal theory by Kalbasi et al. [21]. Song et al. designed a type of wavy-fin of compact heat exchangers by constructal theory to find the optimal configuration [22]. Application of constructal theory has been widely used in the heat exchangers by many researchers. Constructal optimizations of H- and X-shaped heat exchangers were carried out by Chen et al. to maximize the thermal efficiency as optimization objective [23]. A general optimization design method using constructal theory was presented for heat exchanger design by

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Nomenclature

$A_{b,f}$	fin base area (m^2)
A_c	fin cross-sectional area (m^2)
A_s	Convection heat transfer surface area (m^2)
A_{unf}	surface area of unfinned section (m^2)
$A_{b,tot}$	total base area (m^2)
a_1	self-confidence in PSO algorithm (–)
a_2	awarm-confidence factor in PSO algorithm (–)
D	total system diameter $(d + 2L_1 + 2L_2)$ (m)
D_{max}	maximum allowable system diameter (m)
d	tube outside diameter (m)
h	convection heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
k_f	fin thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L_1	fin first part length (m)
L_2	fin second part length (m)
M_{tot}	total mass of fin array (kg)
N_b	number of fin branch in the second part (–)
N_f	total number of fin in the fin array (–)
P	fin cross-section perimeter (m)
q_{conv}	rate of convection heat transfer (W)
q_{tot}	total rate of heat transfer in the base and fin array (W)
q_x	rate of conduction heat transfer (W)
t_1	fin thickness in the first part (m)
t_2	fin thickness in the second part (m)
T	temperature (°C)
T_b	temperature of base surface (°C)
T_∞	environment temperature (°C)
W	fin depth (m)

Greek abbreviation

α_1	$A_{b,f}/A_{b,tot}$ in the $x = d$ (–)
α_2	$A_{b,f}/A_{b,tot}$ in the $x = D$ (–)
θ	$T - T_\infty$ in the first part of fin (°C)
θ_b	$T_b - T_\infty$ in the first part of fin (°C)
β	$T - T_\infty$ in the second part of fin (°C)
ω	inertia factor in PSO (–)
ρ_f	fin density (kg/m^3)

Subscript

f	fin
1	first part of fin
2	second part of fin
b	fin base
tip	fin tip
tot	total
conv	convention heat transfer

Yang et al. to minimize the total cost [24,25]. Azad and Amidpour optimized a shell and tube heat exchanger using the constructal theory to decrease the total cost [26]. Rocha studied the convection in the channels and porous media using constructal design [27]. Some researches were carried out on constructal theory as well as multi-objective constructal optimization and their applications by Chen et al. [28–30].Finally the multi-objective optimization of a pin fin was performed to determine the optimal fin geometry using the genetic algorithm by Hajabdollahi et al. [31].

In the present work, the design and analysis of a cylinder in which its heat transfer surface area is increased by fins has been performed by using the concept of constructal theory proposed by Bejan. Three configurations are studied; three branches fin, two branches fin and simple fin. These configurations are designed and optimized in the way that the rate of heat transfer is maximized as well as the volume of material applied in the fins is minimized. In the other words, the optimization is done by considering the total heat transfer rate and fin material as two objective functions, and the results for the three studied configurations are discussed. For this reason five parameters are selected as the decision variables.

2. Thermal modeling

Schematic of the cylinder and fins array is shown in Fig. 1a. In addition, a cylinder with the constructal fins is also shown in Fig. 1b.

The modeling in the first and second parts of fin is performed under the following assumptions:

1. Steady-state conditions.
2. One-dimensional conduction along the fin.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Constant and uniform convection and conduction heat transfer coefficients.
6. Constant fin cross-sectional area in the both parts.

Applying the conservation of energy requirement to the differential element shown in Fig. 1c, we obtain:

$$q_x = q_{x+dx} + dq_{conv} \tag{1}$$

From Fourier's law we know that [32]:

$$q_x = -k_f A_c \frac{dT}{dx} \tag{2}$$

where A_c is the cross-sectional area and k_f is fin thermal conductivity. Since the conduction heat rate

at $x + dx$ may be expressed as:

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \tag{3}$$

it follows that:

$$q_{x+dx} = -k_f A_c \frac{dT}{dx} - k_f \frac{d}{dx}\left(A_c \frac{dT}{dx}\right)dx \tag{4}$$

the convection heat transfer rate may be expressed as:

$$dq_{conv} = h dA_s (T - T_\infty) \tag{5}$$

where dA_s is the surface area of the differential element. Substituting the foregoing rate equations into the energy balance, we obtain:

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k_f}\frac{dA_s}{dx}(T - T_\infty) = 0 \tag{6}$$

For the studied fins, A_c is a constant and $A_s = Px$, where A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly, with $dA_c/dx = 0$ and $dA_s/dx = P$, Eq. (6) reduces to:

$$\frac{d^2T}{dx^2} - \frac{hP}{k_f A_c}(T - T_\infty) = 0 \tag{7}$$

Considering the $\theta(x) = T(x) - T_\infty$, the general solution of the above equation is:

$$\theta(x) = T(x) - T_\infty = c_1 e^{mx} + c_2 e^{-mx} \tag{8}$$