International Journal of Heat and Mass Transfer 115 (2017) 833–841

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Measuring thermal conductivity of soils based on least squares finite element method

Biao Li [a], Weiya Xu [a], Fuguo Tong [b,*]

[a] College of Civil and Transportation Engineering, Hohai University, Nanjing 210098, China
[b] College of Hydraulic & Environmental Engineering, China Three Gorges University, Yichang 443002, China

ARTICLE INFO

Article history:
Received 25 February 2017
Received in revised form 6 August 2017
Accepted 17 August 2017

Keywords:
Thermal conductivity
Least squares finite element method
Non-iteration
Water content

ABSTRACT

Soil thermal conductivity is an important parameter in the thermal simulations of geo-environmental structures. It is, however, difficult to measure the thermal conductivity as it is affected by several parameters, especially the soil water content. The objective of this paper was to present a measurement methodology based on least squares finite element method (LSFEM). The LSFEM algorithm was developed to directly solve the heat conduction equation, in which the measured temperatures served as input data and the thermal parameters were the unknown variables. The appeal of the LSFEM algorithm lied in its high efficiency and accuracy because no iterations were needed. To obtain the temperature data used for LSFEM solution, we designed a laboratory experiment to produce measurable temperature gradients. The spatial and temporal differences in the temperature measurements were identified to ensure the uniqueness and nonsingularity of the LSFEM solution. The corresponding water content can be simultaneously obtained from the methodology. The measured results suggest that the proposed methodology is promising for measuring the soil thermal conductivity.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Soil thermal conductivity is an important parameter in thermal simulations for the applications in geo-environmental engineering [1,2], including geothermal energy extraction [3,4], radioactive waste repositories [5,6] and ground modification techniques employing heating and freezing [7,8]. The thermal conductivity varies with changes in soil properties, such as the mineral composition, pore fluid, geometrical configuration of pores, and temperature. The variation can reach a high level of complexity, which causes difficulty in measuring the thermal conductivity of soils. However, the internal temperature, which is easy to obtain, can be used to determine the variable thermal conductivity using appropriate analysis techniques.

The rise and fall of internal temperatures submitted to temperature gradients are usually measured in the laboratory. The laboratory techniques used to measure the temperatures can be classified into two main categories. The first involves steady state methods [9,10] in which the flow flux through the sample reaches a constant level. The stabilized temperature distribution is then used to estimate the constant thermal conductivity. Therefore, steady state methods cannot obtain the variations of thermal conductivity. The second category involves unsteady methods [1,11,12] which measure the temperatures during an unsteady heat transfer process. Note that the thermal conductivity is easily burdened with the measurement errors in temperatures that are very difficult to assess. The precision of the experimental measurements depends on the identification of temperature differences.

Calculation of thermal conductivity from the temperatures measured in a medium usually involves the nonlinear inverse heat conduction problem [13]. Most of the analysis methods used to estimate the thermal conductivity are based on optimization techniques [14–19]. In these methods, the unknown thermal conductivity is found by minimizing the difference between measured temperatures and the corresponding results, which are calculated by the heat conduction solution for the given boundary and initial conditions. These methods often require a large number of iterations and forward calculations, and they are usually either locally optimal or inefficient. To avoid numerous iterations, many non-iteration methods [20–25] have been proposed for estimating thermal conductivity with high efficiency. However, some prior assumptions on the approximated function of thermal conductivity or temperature distribution are usually required, and most of these methods are developed for analytical solutions. Because of the complexity associated with the determination of variable

* Corresponding author.
E-mail address: tfg@ctgu.edu.cn (F. Tong).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.08.056
0017-9310/© 2017 Elsevier Ltd. All rights reserved.

thermal conductivity, applications of the non-iteration methods are limited.

In this paper, a new methodology based on least squares finite element method (LSFEM) is proposed for measuring the thermal conductivity of soils. The LSFEM is a non-iteration algorithm for estimating the thermal conductivity from measured temperatures. No prior information on the functional forms of thermal conductivity is required. To ensure the LSFEM solution is unique and nonsingular, an experiment is designed to record the spatial and temporal differences in the internal temperatures. The thermal conductivity and water content of soils can be obtained simultaneously with the proposed methodology.

In the following sections, the LSFEM algorithm is developed first for estimating thermal conductivity from temperature data. The experimental process to measure the temperature data is then described, followed by determination of the thermal conductivity in the laboratory test. Main conclusions from this study are given in the final section.

2. LSFEM algorithm

The governing differential equations for heat conduction problem can be described using Eqs. (1a)–(1d).

$$\rho c\frac{\partial T}{\partial t} + \nabla\cdot(-k\nabla T) = \dot{Q}_v, \quad \text{in } \Omega,\ t\in[0, t_{\max}] \tag{1a}$$

$$T(x,y,z,t) = T_g(x,y,z,t), \quad \text{on } \Gamma_g,\ t\in[0,t_{\max}] \tag{1b}$$

$$-k\frac{\partial T(x,y,z,t)}{\partial n} = q_h(x,y,z,t), \quad \text{on } \Gamma_h,\ t\in[0,t_{\max}] \tag{1c}$$

$$T(x,y,z,0) = T_0(x,y,z), \quad \text{in } \Omega \tag{1d}$$

where ρ, c and k represent density, heat capacity and thermal conductivity, respectively, $\dot{Q}_v$ is rate of heat source per unit volume, T is temperature, and t represents time index. The boundary conditions include the temperature T_g and the heat flux q_h on the subsets Γ_g and Γ_h, respectively, and T_0 is the initial temperature distribution in the control volume Ω. The LSFEM algorithm is used to determine the thermal conductivity and heat capacity from a given temporal and spatial distribution of temperature data.

2.1. Finite element method (FEM) for discretization

The differential Eqs. (1a)–(1d) are not amenable to analytical solutions. Finite element method (FEM) for spatial discretization and finite difference method (FDM) for temporal discretization are adopted to solve the equations in engineering applications.

The Galerkin FEM [26], a weighted residual method in which trial functions themselves serve as weighting functions, is used for spatial discretization, and the one-dimensional finite difference scheme is used for temporal discretization. The finite element approximation of Eqs. (1a)–(1d) can be obtained as

$$\begin{aligned}&[\mathbf{C}(\rho c)]\left(\frac{1}{\Delta t}(\{\mathbf{T}\}^{t_m+\Delta t} - \{\mathbf{T}\}^{t_m})\right)\\ &\quad + [\mathbf{K}(k)]\left(\eta\{\mathbf{T}\}^{t_m+\Delta t} + (1-\eta)\{\mathbf{T}\}^{t_m}\right)\\ &\quad = \{\mathbf{F}\}\end{aligned} \tag{2}$$

where $\eta = (t - t_m)/\Delta t$. The coefficient η may take any value from 0 to 1. The value of $\eta = 0.5$ corresponding to the mid-difference (Crank-Nicholson) scheme is adopted in this paper. $\{\mathbf{T}\}$ is the vector of nodal temperatures; $[\mathbf{C}]$ and $[\mathbf{K}]$ are the matrices of heat capacity and thermal conductivity, respectively; and $\{\mathbf{F}\}$ is the vector of nodal loads, expressed as follows:

$$C_{ij} = \sum_{e=1}^{NE}\int_{\Omega_e}\rho c N_i N_j dV \tag{3a}$$

$$K_{ij} = \sum_{e=1}^{NE}\int_{\Omega_e} k\nabla N_i\nabla N_j dV \tag{3b}$$

$$F_i = \sum_{e=1}^{NE}\left(\int_{\Gamma_h^e} N_i q_h dS + \int_{\Omega_e} N_i\dot{Q}_v dV\right) \tag{3c}$$

The matrix $[\mathbf{C}]$ is a function of volumetric heat capacity ρc, and $[\mathbf{K}]$ is a function of thermal conductivity k. Multiplying the element matrices by the respective temperature vectors and reassembling the columns of the obtained vectors according to elemental unknown parameters, yields equations with material parameters as the basic unknown variables.

$$[\mathbf{C}']\{\boldsymbol{\rho}\mathbf{c}\}^t + [\mathbf{K}']\{\mathbf{k}\}^t = \{\mathbf{F}\}, \quad \text{at } t\in[t_m, t_m+\Delta t] \tag{4}$$

where $[\mathbf{C}']$ and $[\mathbf{K}']$ are the coefficient matrices of volumetric heat capacity ρc and thermal conductivity k, respectively, expressed as follows:

$$C'_{in} = \sum_{e=1, I_{\rho c}=n}^{NE}\int_{\Omega_e} N_i N_j \dot{T}_j dV \tag{5a}$$

$$K'_{in} = \sum_{e=1, I_k=n}^{NE}\int_{\Omega_e}\nabla N_i\nabla N_j T_j dV \tag{5b}$$

where the indexes $I_{\rho c}$ and I_k represent the material numbers of volumetric heat capacity and thermal conductivity, respectively. Eq. (4) can be simplified in the matrix form with a number of N_{nodes} equations and N_x^t unknown variables at the time step $t\in[t_m, t_m+\Delta t]$, written as

$$[\mathbf{A}]\{\mathbf{x}\}^t = \{\mathbf{F}\}, \quad \text{at } t\in[t_m, t_m+\Delta t] \tag{6}$$

where N_{nodes} represents the total number of nodes, and N_x^t is the total number of unknown ρc and k at the time step, $N_x^t = N_{\rho c}^t + N_k^t$. $\{\mathbf{x}\}^t$ is the vector of material parameters at the time step, and $[\mathbf{A}]$ is the coefficient matrix, expressed as

$$[\mathbf{A}] = [\,\mathbf{C}' \quad \mathbf{K}'\,] \tag{7a}$$

$$\{\mathbf{x}\}^t = \begin{Bmatrix}(\boldsymbol{\rho}\mathbf{c})^t\\ \mathbf{k}^t\end{Bmatrix} \tag{7b}$$

2.2. Least squares method (LSM)

The second technique used in the paper to solve Eq. (6) is the least squares method (LSM). It should be noted that the number of linearly independent equations should be equal to or more than the number of unknown variables, otherwise Eq. (6) cannot yield a unique solution. The number of effective equations is determined by the temporal and spatial differences in the temperatures. Therefore, the difference should be measured with high precision and good reliability.

The residual vector components are defined as

$$\xi_i = A_{ij}x_j - F_i \tag{8}$$

The problem of searching for the approximate parameters x_j is formulated as that of minimizing a penalty function of the residual vector:

$$f(x) = \sum_{i=1}^{Nnodes}(A_{ij}x_j - F_i)^2 \rightarrow \min \tag{9}$$

Download English Version:

https://daneshyari.com/en/article/4994015

Download Persian Version:

https://daneshyari.com/article/4994015

Daneshyari.com