International Journal of Heat and Mass Transfer 115 (2017) 930–941

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On the effect of conducting lid for natural convection in a horizontal fluid layer

Jae Ryong Lee *

Korea Atomic Energy Research Institute (KAERI), 111, Daedeok-daero 989Beon-gil, Yuseong-gu, Daejeon 34057, South Korea

ARTICLE INFO

Article history:
Received 25 June 2017
Received in revised form 23 August 2017
Accepted 26 August 2017

Keywords:
Natural convection
Horizontal layer
Conducting lid
Effective Rayleigh number
Zig-zag instability

ABSTRACT

In this study, effect of conducting lid attached in the horizontal fluid layer for Rayleigh-Bénard natural convection was discussed. Geometry which was taken into account was horizontal layer of fluid with conducting lid at the bottom. Conducting lid was heated from the bottom wall and the thermal energy was transferred to fluid layer through the solid/fluid interface, and then was cooled down from above. Thermal conductivity ratio varied to investigate the thermal behavior at the solid/fluid interface. Periodic boundary conditions were employed in the horizontal direction to allow lateral freedom for the convection cells. Prior to investigate thermal behavior of fluid layer with conducting lid, pure Rayleigh-Bénard natural convection without conducting body was reproduced. From two-dimensional Rayleigh-Bénard natural convection, an extension of wavelength of circulating roll cell was examined. A two-dimensional solution was obtained using Chebyshev spectral multi-domain methodology for different Rayleigh number at which the thermal behavior was evolved from steady state to chaotic pattern. Three-dimensional pure Rayleigh-Bénard natural convection was calculated from relatively low Rayleigh number such as 4×10^3 in order to investigate an evolution of thermal plume undergoing a representative zig-zag instability. In order to compare thermal behavior of fluid layer between with- and without conducting lid, effective Rayleigh numbers, Ra_{eff}, was introduced and 10^6 was applied. A solid lid at the bottom affects the flow pattern in that the flow is restricted to increase the dimensionless thermal conductivity. For three-dimensional simulation, periodic boundary condition was also applied along the span-wise direction which was discretized through a Fourier serious expansion with a uniform mesh configuration. For high effective Rayleigh number, thermal flow field was captured by visualizing the three-dimensional vortical structure. The flow behavior at the provided effective Rayleigh number showed a coherent pattern regardless of the magnitude of the thermal conductivity ratio.

1. Introduction

Rayleigh-Bénard convection in a horizontal layer of fluid confined between two parallel plates, with the bottom plate heated and the top plate cooled has been well investigated for decades. It has been established that horizontal fluid layer becomes unstable above a Rayleigh number of 1708 which is known to critical Rayleigh number when isothermal boundary conditions are imposed. Consequently, convective motion sets in the form of steady convective rolls with an aspect ratio (width to height) of about 2 [1]. Numerical models of this problem typically employ a finite computational domain along the horizontal directions. In the interest to approximate an infinite horizontal layer of fluid as closely as possible, periodic boundary conditions are applied at the side boundaries. The advantage of adopting periodic boundary conditions is that it preserves the translational invariance of Rayleigh-Bénard convection along the horizontal directions. The horizontal extent of the computational domain, however, introduces an artificial length scale into the problem, whose influence has been the subject of recent investigations [2,3].

The aspect ratio (width to height ratio) of the computational domain has no influence in the pure conduction regime. In the convective regime the effect of finite aspect ratio is to quantize the possible wavelengths of the convective roll cells. Thus the choice of aspect ratio has an influence on the onset of convective instability observed in numerical simulation. Typically the aspect ratio is chosen to be an integer multiple of the wavelength of the most unstable linear instability mode [4,5].

* Address: Division of Thermal Hydraulics and Safety Research, Korea Atomic Energy Research Institute, 111, Daedeok-daero 989Beon-gil, Yuseong-gu, Daejeon 34057, South Korea.

E-mail address: jrlee@kaeri.re.kr

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.08.086
0017-9310/

A fundamental geometry for natural convection can be implied to one of the conducting boundaries with infinite thermal conductivity, which are located symmetrically on the top and bottom. From a practical viewpoint, natural convection with a finite conducting boundary has been investigated both experimentally and numerically. From the beginning of these researches, onset of thermal convection owing to the effect of the conducting boundary was focused [6]. In addition, theoretical approaches were discussed in conjunction with the effect of a conducting boundary with various thermal conductivities in the thermal convection [7,8]. Recently, natural convection with a conducting boundary of finite thermal conductivity symmetrically located at the top and bottom was conducted [9], in which the conducting boundary affects dynamics of convection and the horizontal platform as decreasing of the thermal conductivity. Upon decreasing the thermal conductivity ratio between a solid and fluid, a temperature field decouples to some extent from the velocity field, and thus more general solutions than just simple periodic rolls become possible. In addition, a scaling analysis for geometry with conducting lid at the top was shown to be compatible with results from two-dimensional numerical simulations that solve full conservation equations [10]. On the other hand, Hunt et al. [11] carried out a research on thermal conduction with a conducting lid at the bottom, in which effects of thermal diffusivity of the solid lid were explained in detail in terms of plume motion and the lid thickness itself. The other geometries have also been investigated to present the effects of conducting wall under buoyancy-driven convection phenomena [12,13]. Recently, Lee [14] has visualized vortical structure of the turbulent thermal plume in a horizontal layer with conducting lid having various thermal conductivity ratio. He have found that by introducing effective Rayleigh number, the flow structure showed a coherent pattern regardless of magnitude of the thermal conductivity ratio.

However, few studies for taking into account the conducting wall are reported in terms of both flow dynamics and turbulent statistical comparison about the existence of conductor at hot wall. In this study, effect of existence of conductor is investigated. Here, a horizontal layer of a fluid heated from below and cooled from above is considered in this study. An aspect ratio (width to height) of π for a horizontal fluid layer is taken into account, and a periodic boundary condition along the horizontal direction is employed to allow lateral freedom for the convection cells. Pure Rayleigh-Bénard convection is reproduced in finite computational geometry. In order to see the flow instability, calculation has been conducted from low Rayleigh number. Prior to three-dimensional calculation, two-dimensional simulation is performed to estimate the wavelength increase as increasing of buoyant force. In addition, the effect of existence of conductor at bottom is investigated. Conducting lid is located at the bottom of the fluid layer with height of 0.3L [14]. In this study, effect of the conducting lid is studied for high Rayleigh number at which an unsteady flow behavior can be expected. The impact of the existence of a lid on the flow structure, dynamics, and overall heat transfer has been evaluated. Especially, vortical structures are visualized using swirl strength as well as conventional turbulent statistical results such as power spectra and autocorrelation.

2. Numerical methodology

2.1. Governing equation and numerical discretization

The system consists of a horizontal layer of fluid heated from below the solid lid at the bottom and cooled from above. A schematic of the three-dimensional geometry along the solid lid at the bottom is shown in Fig. 1. The fluid layer has height L and the solid lid has a height of 0.3L. The bottom wall of the lid is kept at a constant high temperature of T_h, whereas the top wall of the fluid layer is kept at a constant low temperature of T_c. The aspect ratio (width to height) is chosen to be πL. Along the horizontal direction, a periodic boundary condition is enforced. The fluid properties are also assumed to be constant, except for the fluid density in the buoyancy term, which follows the Boussinesq approximation. The gravitational acceleration acts in the negative y-direction.

The continuity, Navier-Stokes, and energy equations in their non-dimensional forms are taken into account as:

$$\nabla \cdot \mathbf{u} = 0 \tag{1a}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + Pr\nabla^2 \mathbf{u} + RaPr\theta \mathbf{k}_2 \tag{1b}$$

$$\frac{\partial \theta_f}{\partial t} + \mathbf{u} \cdot \nabla \theta_f = \nabla^2 \theta_f \tag{1c}$$

$$\frac{\partial \theta_s}{\partial t} = \alpha \nabla^2 \theta_s \tag{1d}$$

The dimensionless variables in the above equations are defined as

$$t = \frac{t^* \alpha_f}{L^2}, \quad \mathbf{x} = \frac{\mathbf{x}^*}{L}, \quad \mathbf{u} = \frac{\mathbf{u}^* L}{\alpha_f}, \quad P = \frac{P^* L^2}{\rho_f \alpha_f^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \alpha = \frac{\alpha_s}{\alpha_f} \tag{2}$$

In the above equations, ρ_f represents dimensional density and α is the thermal diffusivity ratio between a solid lid and fluid layer. The superscript $*$ in Eq. (2) represents the dimensional variables. In addition, $\mathbf{u}$, P, t, and θ are the non-dimensional velocity, pressure, time and temperature. Kronecker delta vector, k2 becomes 1 for y-axis which is gravitational direction. Conventional non-dimensional parameter can be obtained in momentum equation (1b) as follows:

$$Pr = \frac{\nu}{\alpha_f} \text{ and } Ra = \frac{g\beta L^3 (T_h - T_c)}{\nu \alpha_f} \tag{3}$$

where ν, g, and β are the kinematic viscosity, gravitational acceleration, and volume expansion coefficient, respectively. In this simulation in conjunction with having solid lid, it is necessary to introduce new non-dimensional parameter to take into account the effect of the solid lid. By following Lee [14], effective Rayleigh number is defined as

$$Ra_{eff} = \frac{g\beta L^3 (T_i - T_c)}{\nu \alpha_f} \text{ or } Ra_{eff} = Ra \times \langle \bar{\theta}_i \rangle \tag{4}$$

where T_i and $\langle \bar{\theta}_i \rangle$ are the dimensional interface temperature and non-dimensional time- and surface-averaged temperature at the interface between the solid lid and fluid layer, respectively. In this study, high Ra_{eff} number in which turbulent flow behavior happens to be captured is considered varies.

For the boundary conditions, $\theta = 1$ at bottom wall of solid lid and $\theta = 0$ at top of the fluid layer. At the solid/fluid interface, thermal gradient can be conserved by multiply thermal conductivity ratio as follows:

$$\frac{\partial \theta}{\partial \mathbf{n}} = k \frac{\partial \theta_s}{\partial \mathbf{n}} \tag{5}$$

where k (=k_s/k_f) is a thermal conductivity ratio of a solid body to a fluid, and $\mathbf{n}$ is a vector normal to the solid surface.

A spectral multi-domain methodology [15] is used for the spatial discretization. In this technique the overall computational