International Journal of Heat and Mass Transfer 115 (2017) 991–1003

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Laminar forced convection of power-law fluids in the entrance region of parallel plates ducts

Damián Crespí-Llorens [a,*], Nicolas Galanis [b]

[a] Dep. Ing. Mecánica y Energía, Universidad Miguel Hernández de Elche, Av. de la Universidad s/n, 03202 Elche, Spain
[b] Génie mécanique, Université de Sherbrooke, Sherbrooke, QC J1K2R1, Canada

ARTICLE INFO

Article history:
Received 19 June 2017
Received in revised form 31 July 2017
Accepted 27 August 2017

Keywords:
Analytical solution
Entrance length
H condition
Velocity profile
Temperature profile

ABSTRACT

An approximate analytical solution for forced convection of power-law fluids in the entrance region of parallel-plates ducts with the uniform heat flux boundary condition (H condition) is presented and analyzed. It is based on the assumption of similarity between the profiles of the velocity and the temperature in the respective boundary layers and in the fully developed region where exact analytical profiles are obtained from the differential conservation equations. The axial evolutions of the hydrodynamic and thermal boundary layers, of the pressure loss, of the skin friction coefficient and of the Nusselt number are also obtained by applying the integral form of the conservation equations in the entrance region. For a flow behavior index equal to unity (Newtonian fluid) the predicted values of these parameters are in good agreement with corresponding data from the literature.

1. Introduction

Many fluids in the food and petrochemical industries are non-Newtonian. In such applications the determination of parameters such as the friction factor and the Nusselt number is necessary for the calculation of pressure losses and heat transfer rates or temperature distributions. This can be achieved either experimentally or theoretically by solving the appropriate transport equations for typical common geometries (circular ducts, flat ducts, etc.). An important characteristic of these fluids is that they have large apparent viscosities. Therefore, laminar flow conditions occur more often than with Newtonian fluids.

In this paper we analyze the steady-state developing laminar flow of a power-law fluid with constant properties within a parallel-plates duct.

For Newtonian fluids this problem has been solved by several investigators for uniform wall temperature (T condition) and uniform heat flux (H condition). The hydrodynamically developing isothermal flow was solved numerically by Bodoia & Osterle [1] and analytically by Bhatti & Savery [2]. The corresponding thermal entrance problem was solved analytically by Sparrow et al. [3] for the H condition and by Nusselt, Graetz and Lévêque [4] for the T condition. Their expressions for the temperature and Nusselt numbers are given in [4]. According to Shah & Bhatti [4] the most accurate results for the simultaneously developing flow with both thermal boundary conditions are those of a numerical study by Hwang & Fan [5] who presented them in tabular form. For $Pr = 0$ (slug flow) exact analytical expressions for the temperature distribution and the local and average Nusselt numbers are given in [4].

For non-Newtonian fluids this problem has been often studied for isothermal flows. It has also been addressed for forced convection with uniform constant wall temperature (T condition). Thus, Yau & Tien [6] applied the momentum and energy integral method to determine the simultaneous development of velocity and temperature profiles for a constant property fluid obeying the Ostwald-de Waels model (power law). The inlet temperature and velocity profiles were assumed uniform and they obtained approximate expressions for the non-dimensional temperature, velocity and pressure drop as well as for the Nusselt number. The values of the constants appearing in these expressions depend on the flow behavior index and were determined numerically but are not included in [6]. It should be noted that at $x = L_{hy}$ the approximate velocity profile determined by Yaw & Tien does not match the exact analytically determined fully developed velocity profile. Richardson [7] extended the Lévêque solution for hydrodynamically developed flow in ducts with constant wall temperature for the case of a power law fluid. The effect of heat generation by viscous dissipation was included. Matras & Nowak [8] developed a transformation method which converts the isothermal flow of a power-law fluid to an equivalent pseudo-Newtonian flow. They then used the momentum integral method to solve the

* Corresponding author.
E-mail address: dcrespi@umh.es (D. Crespí-Llorens).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.08.093
0017-9310/

Nomenclature

b	half-distance between plates [m]
C_f	skin friction coefficient
c_p	specific heat [J kg^{-1} K^{-1}]
D_h	hydraulic diameter, $D_h = 4b$ [m]
k	conductivity [W m^{-1} K^{-1}]
L_{hy}, L_{th}	hydrodynamic, thermal development lengths [m]
L^*_{hy}	non-dimensional hydrodynamic entrance length, $L^*_{hy} = L_{hy}/(D_h Re)$
$\widehat{L}_{th}$	non-dimensional thermal entrance length, $\widehat{L}_{th} = L_{th}/(D_h Re Pr)$
m	fluid consistency coefficient [Pa s^n]
$\dot{m}$	mass flowrate [kg s^{-1}]
n	flow behavior index
P	pressure [Pa]
P^*	non-dimensional pressure, $P^* = (P_0 - P(x))/(0.5\rho U_0^2)$
Pr	Prandtl number, $Pr = c_p m U_0^{n-1}/(k D_h^{n-1})$
$\ddot{q}_w$	wall heat flux [W m^{-2}]
Re	Reynolds number, $Re = \rho U_0^{2-n} D_h^n / m$
T	temperature [K]
T_0	inlet temperature [K]
U_0	inlet velocity [m s^{-1}]
U_{max}	velocity at $y = 0$ in the hydrodynamically developed region [m s^{-1}]
$U(x)$	velocity in the core region of the developing flow [m s^{-1}]
u, v	velocity components [m s^{-1}]
x, y	Cartesian coordinates [m]
x^*	non-dimensional axial position $x^* = x/(D_h Re)$
$\hat{x}$	non-dimensional axial position $\hat{x} = x/(D_h Re Pr)$
z	non-dimensional boundary layer thickness $z = \delta/b$

Greek letters

Δ, Δ_{th}	thickness of core region for hydrodynamic, thermal problems [m]
δ, δ_{th}	boundary layer thickness for hydrodynamic, thermal problems [m]
ϵ	$= (n+1)/n$
θ	non-dimensional temperature, $\theta = (T - T_0)/(\ddot{q}_w b/k)$
ρ	density [kg m^{-3}]
σ	stress [Pa]

Subscripts

b	bulk
fd	fully developed

hydrodynamic entry problem for the pseudo-Newtonian flow and obtained a single correlation (independent of the flow behavior index) between the pressure drop and the axial distance which agrees well, for $Re > 500$, with experimental data from the literature. Cotta and Özişik [9] used the sign-count method to solve the thermal entrance region heat transfer for laminar forced convection of power-law fluids inside a circular tube and parallel plate channel with the T condition. They presented the local Nusselt number for the entrance region for fluids with values of the flow behavior index $n = 1/3; 1; 3$. Magno et al. [10,11] used the generalized integral transform technique to solve numerically the boundary layer equations for simultaneously developing laminar flow of power-law fluids in a parallel plates channel with the T condition as well. They presented the bulk temperature and Nusselt number at different axial positions along the channel for various flow behavior indices and apparent Prandtl numbers. They noted that for flow behavior indices greater than unity the convective effects near the wall diminish and result in lower values for the Nusselt number in the entrance region. In the thermally developed region they found that, for a fixed value of the apparent Prandtl number, the Nusselt number is essentially independent of the flow behavior index. Gupta [12] applied he transformation method by Matras & Nowak [8] to hydrodynamically developing isothermal flows of power-law fluids in circular pipes and parallel-plates ducts and compared the results of four integral approaches used to solve the equivalent pseudo-Newtonian flow. Recently, Galanis & Rashidi [13] obtained a new solution for the Graetz problem extended to power-law fluids and mass transfer with phase change at the walls. The temperature and concentration spatial distributions were used to illustrate the effects of the fluid nature on the axial evolution of the sensible and latent Nusselt numbers as well as on the local entropy generation due to velocity, temperature and concentration gradients.

Our literature search has yielded few studies for forced convection of power-law fluids in ducts with the H condition (constant uniform heat flux). Cotta & Özişik [14] addressed this problem for hydrodynamically developed flows in circular tubes and parallel-plate ducts and derived exact expressions for the local and average Nusselt numbers. However, to the best of our knowledge, the simultaneously developing hydrodynamic and thermal flow of power-law fluids in ducts with the H condition has not been treated. In view of this situation the present study was undertaken. It presents approximate analytical expressions for the velocity and temperature distributions in the entrance region of the duct which match the corresponding exact expressions at L_{hy} and L_{th} respectively. These expressions are used to calculate the axial evolution of the friction coefficient and the Nusselt number for different fluids.

2. Statement of the problem

We consider the two-dimensional flow of an incompressible power-law fluid with constant properties between two parallel plates (Fig. 1). A constant uniform heat flux is applied at the two solid-fluid interfaces. At the duct inlet ($x = 0$) the velocity and temperature of the fluid are constant and uniform (respectively U_0 and T_0 with the former parallel to the plates). Therefore this is a steady-state forced convection problem, i.e. the hydrodynamic field does not depend on the temperature. In Fig. 1 δ and δ_{th} are the

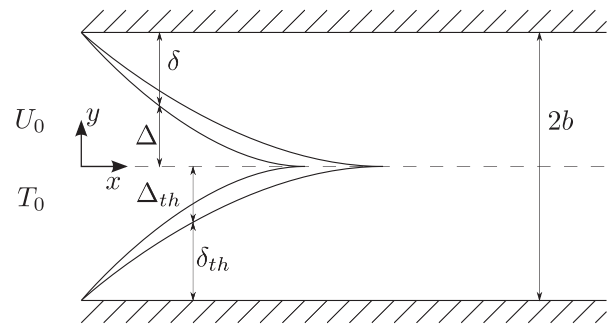

Fig. 1. Schematic configuration of the problem under study.