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A computational approach to the modeling of the glaciation of sea offshore gas pipeline

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ABSTRACT

In the present study a computational approach to the modeling of the glaciation of sea offshore pipelines completely exposed to the seawater is proposed. The phase transform "seawater – ice" is taken into account and Stefan condition on the phase change boundary is considered. The mathematical model is based on Dirichlet problem for the nonlinear heat equation in the domain with fixed boundaries. The model is obtained by transformation of classical unsteady Stefan problem in the ice layer, which is considered as domain with unknown moving boundary. The calculations based on the proposed model may be realized by the implicit methods, which are convenient for numerical solution of two- and three-dimensional problems. The temperature of the gas flow and thermal layers of the pipe is input as a time function.

The finite-element formulation of the Dirichlet problem for the nonlinear heat equation is proposed. The iterative method for the practical calculations based on the solution of the linear problem on every iteration is considered. The method does not use Jacobi matrix. Algorithm of the method may be easily realized on modern programming languages and software packages.

Some problems of the practical realization of the implicit method are solved. The methods of the estimation of the length of heat boundary layer and Dirac delta function definition interval are proposed. Several types of the approximating functions for the Dirac delta function and the coefficient of the thermal conductivity are considered. For one-dimensional problem, it is shown that the piecewise constant approximations are preferable. The program of the algorithm of the implicit method for a solution of two-dimensional problems is realized in the FreeFem++ software package in the case of fixed mesh.

Two model problems are considered: the problem of the uniform glaciation of the pipe and the problem of weakly asymmetric glaciation with a model profile of the initial ice layer.

1. Introduction

Nowadays pipelines are a widely-used and convenient way of transporting of natural gas. Gas flow in pipes takes place at high pressures. During the transportation through the pipeline the gas cools down due to the heat exchange with the surrounding media and some gas dynamic effects. In northern seas for the cases of long sea gas pipelines without compressor stations (such as gas pipeline from the Stokman gas field in the Barent Sea) gas temperature in the pipeline may be lower than the seawater – ice transition temperature and the glaciation become possible [1]. Growing ice layers affects on the heat exchange processes between the gas and surrounding media, as well as on the pipeline buoyancy [1]. The density of the part of the pipeline structure in case of glaciation may change, that may lead to the detachment of the envelope of the pipe and to ruptures. A rupture of the pipeline can cause serious accidents as well as economic losses and must be avoided [2]. So for the prediction of emergency situations the dynamics of the glaciation should be estimated. At the next stage of the investigation the stress state in the glaciated pipeline may be analyzed. Therefore, the construction of mathematical models of the glaciation process as a base for simulation tools is required.

Mathematical models of natural gas flows in pipelines play an important role in designing, monitoring and operating of natural gas pipelines. These models aim to predict the measurable quantities such as pressure, density, temperature and mass flow. Due to the length of the pipeline, in most cases the model is considered as one-dimensional and based on the mass, momentum and energy conservation laws. Sufficiently complete models of nonisothermal gas flows in high-pressure gas transmission pipelines are

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presented at [3–8]. The models of two-phase flows are presented in [9,10].

The problem of the heat transfer in the pipe and surrounding media is considered in [3,11–13]. In [11] a model for the steady state heat transfer between the gas and pipeline surroundings is considered. In [3] the unsteady axis-symmetric heat transfer is modeled by a so-called element method based on equations representing thermal balances of the coaxial cylinder layers. For the purpose of heat transfer discretization of this domain is realized. It is proposed that every element has the same thermal conductivity. The process of heat transfer may be modeled by a minimum two cylindrical layers as heat capacitors. The results of the simulations demonstrate that unsteady heat transfer model better predicts the behavior of the gas characteristics in the pipelines, whereas steady-state model overestimates the amplitude of the temperature fluctuations. In [13] the heat transfer between pipe of the offshore buried pipeline and surroundings is modeled. The differences in outlet temperatures between steady model and experimental measurements are quantified. It is shown for steady model that there exists a discrepancy of 1–2 K between the measured and modeled outlet temperature. This difference may be significant, when the temperature of the gas is closer to 271 K and glaciation may occur. In [12] two models are compared for the cases of onshore and offshore pipelines. The results for the onshore buried pipeline demonstrate that steady model over predicts the amplitude of temperature changes. It must be noted, that in northern seas (such as the Barent Sea) the pipelines are completely exposed to the seawater and buried in the sea bottom only near the coast. So for these case the models of heat exchange for the offshore pipelines should be considered. For the correct modeling of heat transfer around the pipe in the case of long sea offshore gas pipeline, the possibility of the glaciation of the pipe must be taken into account. One-dimensional model of the axis-symmetric glaciation of the pipe, which is not taking into account in previous studies, is presented by Kurbatova and Ermolaeva in [1,14]. In these works the case of cross-section of offshore pipeline, completely exposed to the seawater, is considered. The front-tracking method, which is convenient to use only in 1D case, is proposed. This model can be extended to 2D and 3D cases. In these situations the problem of transformation of this model for the convenient implementation of computations in widely-used software packages is appeared. This problem is solved in the presented paper.

The glaciation process may be modeled by initial-boundary problem for the heat equation in the domain with moving unknown boundary, where the water-ice transition is realized (phase change boundary, PCB). On the PCB's the Stefan condition is considered [15]. There are two main approaches for the numerical solution of Stefan problem, based on the description of the PCB.

The first approach is based on the *explicit* description of PCB, where marker nodes of the computational grids, which lie on this boundary, are selected and treated explicitly. The approach is based on the fixed or adaptive grids or meshes. If the grid or mesh is fixed, PCB is tracked by special approaches. If adaptive grids or meshes are considered, they are generated at the initial time moment in such a way as markers lie on the PCB. These markers at every time step are displaced, according to the numerical solution of Stefan problem. According to this idea, initial grid is distorted (mesh is deformed) and a new grid must be generated or corrected. The typical realization of explicit approach is a front-tracking method. In [16] method with variable time step and fixed grid, when the PCB move from one step to another is proposed. Murray and Ladis in [17] provide approach based on space grid with variable step, but with fixed number of space intervals between fixed and moving boundary. Crank in [18] discuss various finite-difference methods with variable time steps and a change of space variable to fix the moving boundary. Ermolaeva and Kurbatova in [14] realize a front-tracking method proposed in [19] by Vasiliev to 1D problem of gas pipeline glaciation modeling. As it is mentioned in [20], these methods are effective only in 1D case, extension to 3D and 2D problems lead to various difficulties.

In *implicit* approach, computational grid is usually fixed and the position of PCB is obtained indirectly. An example of such approach is a continuous method [21], which is based on the transition to the problem for the nonlinear heat equation in domain with fixed boundaries. This problem is usually written for the enthalpy [15], which is bijectively related to the temperature. Some approaches use enthalpy together with temperature [22], some problems are formulated on the base for the problem for enthalpy, but for temperature as a dependent function [21,23,24], in these cases the nonlinear equation only for the temperature is considered. In this formulation the flux condition on the PCB is automatically satisfied and PCB appears as a curve or surface of a jump discontinuity of the enthalpy. PCB is defined by grid points, where the equality of the numerically obtained temperature to the phase transition temperature is realized. Approaches based on implicit methods are quite flexible and may be applied in multi-dimensional case, in domains with complex boundaries and convenient for common discretization techniques.

The approaches to discretization of Stefan problem are based on standard techniques, such as finite-difference method, boundary element method and finite-element method (FEM) with fixed or adaptive grids. FEM may be considered as a much flexible due to its possibilities for realization in widely-used software packages for simulation. A general reviews of FEM application to Stefan problem are considered in [20,25]. Feulvarch et al. in [22] proposed method based on the mixed enthalpy-temperature model with fixed grid technique. The method is based on the finding of two unknown variables—enthalpy and temperature. There is a problem with the computation of enthalpy on the PCB, where the discontinuity of this function takes place. The presence of discontinuities may lead to some numerical problems (such as fictitious oscillations) in numerical solutions. Temperature is not computed directly, but is recalculated from the values of enthalpy, so the additional stage of the computational algorithm on every time step is added. Implicit time discretization based on the implicit Euler method is considered. Equations obtained after discretization are solved by Newton—Raphson method. Despite the fast convergence, this method needs the computation of the Jacobi matrix on every iteration. For the cases of meshes with large number of nodes it leads to the computation and storage of the big data massive on every iteration. In [26] hybrid method based on FEM and meshless element free Galerkin method is proposed for phase change problems with natural convection. The governing equations are solved by a meshless method near the phase transition front, while in regions away from this front problem is solved by FEM. Extended FEM (XFEM) is realized to capture the transient solution and motion of phase boundaries on a fixed mesh without adaptive remeshing in [27]. In [28] XFEM is proposed for a Stefan problem in the level set formulation. The common idea of methods based on XFEM is to explicitly track the PCB and to construct an enriched finite element spaces depending on the interface position while keeping the underlying mesh fixed. Additional basis functions are designed to capture the phase change across the PCB, i.e., they are continuous but their first derivatives have a jump. Unfortunately, this method is not realized in GNU software, such as FreeFem++. Lun et al. in [29] propose FEM with a Schur complement formulation that uses a linear iterative solver to solve a Stefan problem for steady-state phase change. In [30] FEM with adaptive mesh for convective problems based on Newton method is constructed. The proposed model includes complex effects such as natural convection of water and air, water freezing and phase change processes.