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Review

Does mathematics contribute to the nanofluid debate?

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ABSTRACT

Recent experimental evidence has clearly demonstrated that nanofluids do not provide the greatly enhanced heat transfer predicted in the past. Despite seemingly conclusive proof there is still a great deal of current mathematical research asserting the opposite result. In this paper we scrutinise the mathematical work and demonstrate that the disagreement can be traced to a number of issues. These include the incorrect formulation of the governing equations; the use of parameter values orders of magnitude different to the true values (some requiring nanoparticle volume fractions greater than unity and nanoparticles smaller than atoms); model choices that are based on permitting a reduction using similarity variables as opposed to representing an actual physical situation; presentation of results using different scalings for each fluid.

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1. Introduction

There exists a wide experimental literature concerning the heat transfer properties of nanofluids. In the past remarkable increases in thermal conductivity, viscosity and heat transfer coefficient were reported with the addition of a very small volume fraction of nanoparticles to a base fluid. The experimental work was supported by both theoretical and numerical investigations. However, a remarkable spread in the experimental data prompted a benchmark study by 34 laboratories around the world [1]. One of their main conclusions was that *no anomalous enhancement of thermal conductivity was observed in the limited set of nanofluids tested in this exercise.* This result is backed up by other recent studies. The Stanford NanoHeat Group state that *particle based nanofluids show little promise for heat exchanger design*, although they do suggest it is possible carbon nanotube (CNT) based fluids may be viable [2]. Putra et al. [3] state that with the addition of nanoparticles *a systematic and definite deterioration in natural convective heat transfer has been found to occur*. Similar results are reported in [4–7].

Despite very convincing experimental evidence that nanofluids are not the predicted saviours in the heat transfer world there is still a great deal of research activity in this area. One particularly lively area is in the mathematical modelling of nanofluid flow using the system of equations developed by Buongiorno [8]. In particular, practitioners of boundary layer theory, similarity solutions

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and the Homotopy Analysis Method have published thousands of papers dealing with different forms of and extensions to Buongiorno's equations and subject to a variety of boundary conditions. In contrast to the recent experimental results these authors are unanimous in the opinion that nanoparticles have a positive effect on the thermal field and heat transfer characteristics. This is summarised in the review of Haddad et al. [9] who state that theoretical results show that *nanofluids significantly improve the heat transfer capability of conventional heat transfer fluids whereas experimental results showed that presence of nanoparticles deteriorates heat transfer systematically*. In Haddad et al. [10] it is stated that theory shows *there is always an enhancement in heat transfer by the presence of nanoparticles*.

The mathematical analysis of Buongiorno's equations appears to start with the work of Tzou [11] which involves a study of the linear stability of a heated, thin layer of nanofluid. In that paper. Buorgiorno's equations were extended to include buoyancy in the fluid momentum equation. The results showed a decrease in the critical Rayleigh number of two orders of magnitude. In conclusion Tzou stated that *clearly, Brownian motion and thermophoresis of nanoparticles introduce additional nonlinear effects for heat transport in nanofluids*. This is despite a subsequent statement that the effect of nanoparticles is only noticeable for a Lewis number below 10, while accepting that for actual nanofluids the true value is three to four orders of magnitude greater. In fact, as we will see later the value is often even higher.

Tzou's work was followed by a series of papers by Neild and Kuznetsov dealing with nanofluid stability and convective flow [12–14]. In Neild and Kuznetsov [14] they investigate convection driven flow past a vertical plate placed in a porous medium and so write down a modified form of Buongiorno's system to account for buoyancy and porosity. This is perhaps the first published paper where standard boundary layer theory is applied to the nanofluid equations. In [12] they examine the same problem without porosity. Khan and Pop [15] employ the equations provided in [12,14] but investigate a simpler configuration where flow is driven by the movement of a 'stretching surface'. Consequently their system contains no buoyancy terms and in fact more closely follows Buongiorno's original model.

The papers [12,14,15] use standard boundary layer approximations and similarity variables to reduce the problem to a set of ordinary differential equations. They have a high number of citations and consequently a plethora of papers follow employing similar reductions and the same basic form of heat and nanoparticle concentration equation. These extensions and modifications include magnetohydrodynamic effects; radiative heat flux in the heat equation; permeable substrates; heat generation/absorption; non-Newtonian fluids; flow in a cylindrical geometry; flow in a cylindrical geometry embedded in a porous medium; a permeable cone in a porous media; various far-field flow configurations; nanofluids with micro-organisms, see [16–21,23,24]. Simply for the stretching sheet model there are studies with sheets moving at a constant rate, with velocity proportional to distance x; proportional to x^n; proportional to x/t (and then with a substrate temperature proportional to x/t^2); exponentially increasing [16,25–28].

Increasing the model permutations, there is also a wide variety of boundary conditions on temperature (constant, fixed flux, convective, radiative); velocity (no slip, various forms of slip, suction, etc.) [23,29–32]. In the absence of a permeable surface the physically sensible boundary condition for the nanoparticles at the substrate is zero flux, i.e., the particles cannot penetrate the substrate. This was imposed in the original paper of Buongiorno [8]. However, the early mathematical studies and many recent ones specify the particle concentration at the substrate [29,33–35]. In [10] it is pointed out that this condition is 'somewhat arbitrary' and so they employ the zero particle flux condition. Neild and Kuznetsov [36,37] then use this condition to revisit their previous problems. Tham et al. [35] retain a fixed concentration condition while noting that the zero flux condition or *the 'new' boundary condition of [36] is a more natural (physical) assumption*.

The physical justification for the mathematical configurations invariably relates to extrusion of polymer sheets and filaments, melt spinning, manufacture of plastic and rubber sheets and cooling of large metallic plates [15,38,39]. The actual relation between model and the physical system can be difficult to determine. For example, none of these proposed processes involves an exponentially stretching sheet with a constant temperature moving through a porous medium. In fact the justification for the exponentially stretching sheet studied in [40] is simply a comment in [41] that the velocity may not be linear: [42] then cites [40] as a justification. In [32] slip flow over a stretching sheet is studied. The slip lengths quoted are up to 68 mm, which is an incredibly large number, see [43]. These values come from early experiments on flow in carbon nanotubes [44], which is obviously a completely different physical situation. Further, the values are incorrect, in an erratum to the experimental paper, [45], it is stated that the slip lengths should have been written as microns, rather than millimetres. There is also a general avoidance of actual physical parameter values in the mathematical studies, instead values for the non-dimensional numbers are taken from previous papers, so propagating errors. However, if they are discussed, the nanofluids are standard, such as water or ethylene glycol containing Cu, CuO, Al_2O_3, TiO_2 or Au nanoparticles, see [11,17,25,31] for example. In Raju et al.[46] the study focuses specifically on ethylene glycol with Cu or CuO particles, they also use standard relations to determine the nanofluid properties. So the parameter values should be very similar to those in Buongiorno's original paper and in general are easily obtained from the literature.

The enhanced heat transfer predicted by all of the cited theoretical studies and their many offspring is in direct contradiction to many experimental results. This has led to confusion in the research community. At the end of their experimental study Li and Peterson [6] question the contradiction *the controversy resulted from simulation study and experimental study is still not clear* and go on to discuss possible mechanisms for this. Since the nanofluids discussed are standard it is clear that the disagreement cannot arise due to the choice of fluid. Consequently, the goal of this paper is to determine the cause of the disagreement. We will show unequivocally that it arises from a series of errors and incorrect values used in the mathematical models. The problems will be illustrated by working through a specific flow configuration common in the literature. However, the conclusions are general and not restricted to this simple flow configurations.

2. Mathematical model

The key parameters in this debate relate to the motion due to Brownian diffusion and thermophoresis. The Brownian diffusion refers to the effect the Brownian motion of the base fluid molecules has on the nanoparticles. Thermophoresis (or the Soret effect specifically in liquids) is a related effect whereby the presence of a temperature gradient drives the motion: hot molecules have more kinetic energy than cool ones, so hot base fluid molecules tend to drive the nanoparticles in the direction of lower temperature. The two terms are quantified by the following expressions

$$D_B = \frac{k_B T}{3\pi\mu_{bf} d_p}, \qquad D_T = \frac{0.26 k_{bf}}{2k_{bf} + k_p}\frac{\mu_{bf}}{\rho_{bf}}\phi, \tag{1}$$

where k_B is the Boltzmann constant, T is temperature, μ the dynamic viscosity, d_p the nanoparticle diameter, k the thermal con-