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Influence of chemical reaction and heat source on dissipative MHD mixed convection flow of a Casson nanofluid over a nonlinear permeable stretching sheet

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A B S T R A C T

A mathematical model has been developed to study the mixed convection on MHD flow of Casson fluid over a nonlinearly permeable stretching sheet with thermal radiation, viscous dissipation, heat source/sink, chemical reaction and suction. In this investigation we also incorporated the Buogiorno's type Nanofluid model that include the effect of Brownian motion and thermophoresis. Suitable similarity transformations are used to reduce the governing partial differential equations (PDEs) into a set of non-linear ordinary differential equations (ODEs). These equations are solved using homotopy analysis method (HAM). The convergence of series solutions is discussed explicitly. A comparison has been made and found to be in good agreement with a previous published result on special cases of the problem. The graphical and tabulated results are given to deliberate the physical nature of the problem. Casson parameter is helpful for minimizing the skin friction, the rate of heat and mass transfer. Whereas suction is useful in improving the heat and mass transfer rates.

1. Introduction

The study of nanofluids is one of the thrust areas of contemporary research due to their tremendous potential with respect to enhanced heat transfer. The term 'nanofluid' describes a liquid suspension composed of tiny particles of diameter less than 100 mm. Choi [1] was the first researcher who introduced the term 'nanofluid' to describe a new class of fluid. The phenomenon of thermal conductivity enhancement as the main characteristic feature of nanofluid has been monitored by Masuda et al. [2]. This phenomenon suggests the utilization of nanofluids in advanced nuclear systems [3]. Later on, Buongiorno [4] implemented a new model for nanofluid flow by considering the effect of Brownian motion and thermophoresis. By using this model, Kuznetsov and Nield [5] investigated the natural convection flow of nanofluid past a vertical plate. Earlier, Prabhat et al. [6] presented a comprehensive survey of convective transport in nanofluids, trying to reach a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity. Khan and Pop [7] studied the problem of nanofluid flow over a stretching sheet. Makinde and Aziz [8] obtained the numerical solution for boundary layer flow and heat transfer of nanofluid over a stretching surface with convective boundary conditions. Das [9] analyzed the nanofluid flow over a nonlinear stretching sheet under partial slip conditions. Hady et al. [10] discussed the radiation effect on viscous nanofluid over a nonlinearly stretching sheet. Ahmad and Pop [11] studied the characteristics of mixed convection flow of nanofluids over a vertical flat plate embedded in a porous medium. At the present time there are various papers on the nanofluid flow by considering different effects such as magnetic field, chemical reaction, Brownian motion, heat generation/absorption and so on. Due to aforesaid importance many researchers studied numerically the phenomena of nanofluids considering different flow controlling parameters such as Stretching sheet with [two-phase model] Sheikholeslami et al. [12] [stagnation point and convective conditions] Akbar et al. [13], [Scaling transformation with porous sheet] Hamad and Pop [14], [Buoyancy forces with radiated sheet] Rashidi et al. [15] and [nonlinear porous sheet with dissipation] Sibanda and Khadir [16].

Casson fluid model [17] is a prominent model for many industrial as well as science and engineering fields such as blood,

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Nomenclature

u, v	velocity components in x and y directions
k	thermal conductivity
$\alpha = \frac{k}{(\rho c)_f}$	thermal diffusivity
σ	electrical conductivity
B_0	constant
ν	kinematic viscosity
β	Casson fluid parameter
ρ_f	density of the base fluid
g	acceleration due to gravity
β_T	coefficient of thermal expansion
β_C	coefficient of expansion with concentration
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
Q_0	volumetric heat generation/absorption
C_p	specific heat at constant pressure
T	fluid temperature
C	fluid concentration
k_0	chemical reaction coefficient
v_w	velocity of suction
ψ	stream function
T_w	wall temperature
C_w	nanoparticle concentration
T_∞	ambient value of temperature
C_∞	ambient value of the nanoparticle fraction
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_p$	effective heat capacity of a nanoparticle
n	nonlinear stretching parameter
a	positive constant
$U_w = ax^{n-\frac{1}{2}}$	stretching velocity
$B(x)$	magnetic field
$Gr = \frac{g\beta_T(T_w-T_\infty)}{a^2x^{2n-1}}$	local Grashoff number
$Gc = \frac{g\beta_C(C_w-C_\infty)}{a^2x^{2n-1}}$	local modified grashoff number
$M = \frac{2\sigma B_0^2}{a\rho_f(n+1)}$	magnetic parameter
$Pr = \frac{\nu}{\alpha}$	Prandtl number
$R = \frac{4\sigma^* T_\infty^3}{k^* k}$	radiation parameter
$Nb = \frac{(\rho c)_p D_B(C_w-C_\infty)}{\nu(\rho c)_f}$	Brownian motion parameter
$Nt = \frac{(\rho c)_p D_T(T_w-T_\infty)}{\nu(\rho c)_f T_\infty}$	Thermophoresis parameter
$Ec = \frac{U_w^2}{C_p(T_w-T_\infty)}$	Eckert number
$Q = \frac{2xQ_0}{(\rho c)_f(n+1)U_w}$	heat generation/absorption coefficient
$Sc = \frac{\nu}{D_B}$	Schmidt number
$\gamma = \frac{2xk_0}{(n+1)U_w}$	chemical reaction parameter
$S = -\frac{v_w}{\sqrt{\frac{a\nu(n+1)}{2}x^{(\frac{n-1}{2})}}}$	suction parameter

Subscripts

w	condition at the surface
∞	condition at the free stream

chocolate, honey, etc. Some suspension flows are also closely related to this model. This model exhibits a yield stress. Casson fluid behaves as solid when the shear stress is less than the yield stress and it starts to deform when shear stress becomes greater than the yield stress. Mehta et al. [18] investigated the behavior of Casson fluid under yield stress through a homogeneous porous medium bounded by circular tube. Swati Mukhopadhyay [19] studied flow and heat transfer characteristics of Casson fluid over a nonlinear stretching surface. Mustafa and Khan [20] explored the magnetic field effect on Casson nanofluid over a nonlinearly stretching sheet. Some recent studies concerning the flow and heat transfer analysis of Casson fluid can be found in [21–27].

In this article, we analyzed the mixed convection on MHD boundary layer flow of Casson nanofluid which obtains from the nonlinear stretching of a sheet using HAM [28–30]. In this study we considered suction, magnetic field, thermal radiation, viscous dissipation, heat generation/absorption and chemical reaction.

2. Mathematical formulation

Consider a steady, two dimensional and incompressible mixed convection MHD flow of a Casson nanofluid located at $y = 0$. The flow is confined to $y > 0$ and the sheet is stretched along the x -axis with velocity $U_w = ax^{n-1/2}$, where $n \geqslant 0$ is a nonlinear stretching parameter and $a > 0$ is a constant. In this investigation we also incorporated the Buogiorno's type Nanofluid model that include the effect of Brownian motion and thermophoresis. The fluid is electrically conducted due to an application of magnetic field $B(x) = B_0 x^{(\frac{n-1}{2})}$ normal to the sheet. The magnetic Reynolds number is assumed small and so the induced magnetic field can be considered to be negligible. It is assumed that T_w and C_w are the wall temperature and nanoparticle concentration and as $y \to \infty$, the ambient values of temperature and nanoparticle fraction are T_∞ and C_∞ such that $T_w > T_\infty$ and $C_w > C_\infty$. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is

$$\tau_{ij} = \begin{bmatrix} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi_c > \pi \end{bmatrix}$$

where μ_B is the dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of the fluid, π is the product of the component of deformation rate with itself, $\pi = e_{ij}e_{ij}$, e_{ij} is the $(i,j)^{th}$ component of the deformation rate and π_c is the critical value of this product based on the non-Newtonian model. The velocity field is taken as $V = [u(x,y), v(x,y), 0]$. Under the boundary layer approximations, the governing equations for conservation of mass, momentum, thermal energy and nanoparticle concentration of this problem can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_f}u + g\beta_T(T-T_\infty) - g\beta_C(C-C_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\nu}{C_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} + \frac{1}{(\rho c)_f}\frac{Q_0}{(T-T_\infty)}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_0(C-C_\infty). \tag{4}$$

Subject to the boundary conditions

$$\begin{aligned} &u = U_w = ax^{n-1/2},\ v = v_w,\ T = T_w,\ C = C_w \ at\ y = 0, \\ &u \to 0,\ v \to 0,\ T \to T_\infty, \qquad as\ y \to \infty. \end{aligned} \tag{5}$$