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A new efficient and accurate procedure for solving heat conduction problems

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ABSTRACT

The half-boundary method (HBM) which reduces the order of partial differential equations, is extended with detailed formula derivations to solve heat transfer problems. The method has a comparable accuracy with analytical solution even when a few nodes are involved in calculation. And it also saves more time than finite volume method (FVM). HBM can separately calculate the field variables at any point of interest in the domain without uniform mesh or dimensional length. Variables at any node within the domain are associated with those on one of the two boundaries. For one-dimensional problems, only two-order matrices are calculated in HBM instead of huge-order matrices required in FMV. Therefore, this method shows great potential in accurate and efficient calculating multi-dimensional heat transfer and fluid flow problems with complex geometries and huge grids. In this paper, we introduce the fundamental theory and test the applicability, accuracy and efficiency of HBM by solving six simple one-dimensional problems, one two-dimensional problem and one practical problem of the temperature field simulation of double vessels in China Experimental Fast Reactor (CEFR), which can give a universal sense for more complex problems. ANSYS simulation is also utilized to verify the accuracy of HBM. Matlab codes are used.

1. Introduction

Routinely used numerical methods such as finite difference method (FDM) [1,2], finite volume method (FVM) [3–6] and finite element method (FEM) [2,7,8] are marching their way in solving heat transfer and fluid flow problems both in macroscopic and mesoscopic areas such as heat conduction problems in nuclear reactors [9–12] and nano-fluid convection problems in microelectronic devices [13–17]. Among these methods, FVM has become a mainstream method due to its definite physical interpretation. But FVM produces a high-order matrix, leading to a high workload, especially when large number of grids are involved in calculation. Despite the tremendous developments and achievements in computer capacity, it is still cost inefficient in terms of handling intricate models coupled with complex geometries, variable boundary conditions and multi-physics interactions. And the total processing duration is more important if transient problems are considered.

To improve the computation accuracy, the commonly used technologies are increasing the grid numbers or adopting higher-order terms of partial differential equations to reduce the truncation error. However, these technologies inevitably increase the computation time. Therefore, some researchers chose to increase the grid numbers at certain zone of the domain [18], or deduced the high-order discrete equations for the boundary nodes [19]. But these improvements are only applicable to certain cases and models, and still cannot balance the accuracy and efficiency at the same time. As for the boundary nodes, approximation calculation is still commonly adopted. The accuracy hence cannot be guaranteed if few grids are involved.

As for increasing the calculation efficiency, the modified methods are mostly on the contrary to those utilized to increase the computation accuracy. One aspect is aimed at reducing the effort devoted to mesh generation which would be the most taxing part for a complex model. For example, Langmayr et al. [20] modified the mesh at certain parts of the complex geometrical model and reduced the grid numbers. Kim et al. [21] utilized a double-grid method for efficient computation of a phase-field model.

The other meaningful simplification is to modify the formulae by partially changing the equations [22,23] or totally reducing the order of equations. Compared with the former which usually accounts for particular cases, the reduced order model is more efficient because it can reduce the freedom degrees and convert the calculation of partial differential equations to the calculation of lin-

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Nomenclature

S	heat source term
s	heat source value, W/m^3
T, X_1	temperature, K
V, X_2	heat flux, J/(m^2 s)
W, X_2	heat flux, J/(m^2 s)
k	thermal conductivity coefficient, W/(m K)
h	convective heat transfer coefficient, W/(m^2 K)
c	heat capacity, J/(kg K)
ρ	density, kg/m^3
x	x coordinate, m
y	y coordinate, m
Δx	distance between adjacent nodes in x direction, m
Δy	distance between adjacent nodes in y direction, m
t	time, s
Δt	time step, s

Subscripts
i, $i-1$ grid node

Superscripts
m, $m-1$ time moment

ear equations in a lower-dimensional space. Berntsson [24] achieved the solutions to one-dimensional inverse heat conduction problems. Wang et al. [25] established a reduced order model for steady-state heat convection using proper orthogonal decomposition (POD) and Galerkin projection methods. But the shape functions are necessary in Galerkin methods.

In this paper, HBM [26–29] is extended for solving heat conduction problems, and the detailed equations are derived. It handles the higher-order derivatives as independent variables. In this way, only second-order matrixes are involved in the calculable process of HBM, which avoids the building of complex matrix and arduous matrix inversion that required in FVM. Also, the temperature and heat flux fields can be solved simultaneously. In addition, HBM can solve problems with different boundary types and the problems with two conditions on only one-side boundary, because the calculation marches from one side to the other. No uniform grid or certain node length is required. HBM can calculate the variable fields at any point of interest in the domain without solving the results for all the nodes, which makes it more flexible.

2. Half-boundary method

As a fundamental research, one-dimensional problems including a practical case are intensively studied in this paper. And then the HBM is expanded to a two-dimensional steady-state problem with linear material properties and temperature boundaries.

2.1. One-dimensional HBM

One-dimensional heat conduction equation of HBM in a Cartesian coordinate can be written as follows:

$$\rho(x,T)c(x,T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k(x,T)\frac{\partial T}{\partial x}\right)+s(x,t) \tag{1}$$

where $\rho(x,T)$, $c(x,T)$ and $k(x,T)$ are the density, heat capacity and thermal conductivity coefficients, respectively. For most of materials, the properties are inconstant. In this paper, material properties are simplified to be a function of temperature and location. $s(x,t)$ is the source term, and is assumed to vary with location and time. While for steady-state condition, s is only the function of location. The order of Eq. (1) is decreased by setting $V=k(x,T)\frac{\partial T}{\partial x}$, and then Eq. (1) is simplified as:

$$\begin{cases}\rho(x,T)c(x,T)\frac{\partial T}{\partial t}=\frac{\partial V}{\partial x}+s(x,t)\\ V=k(x,T)\frac{\partial T}{\partial x}\end{cases} \tag{2}$$

For transient problems, the following initial and boundary conditions are adopted:

$$T^0(x)=g(x),t=0 \tag{3}$$

$$T_W(t)=f_1(t),t>0 \tag{4}$$

$$V_W(t)=f_2(t),t>0 \tag{5}$$

$$V_W(t)=h(t)(T_W(t)-T_\infty(t)),t>0 \tag{6}$$

where Eq. (3) is the initial boundary when time = 0 s, g(x) is known. Eq. (4) is the first kind boundary, where T_W is the temperature at the boundary location, $f_1(t)$ is known. Eq. (5) is the second kind boundary, where V_W is the heat flux at the boundary location, $f_2(t)$ is known. And Eq. (6) is the third kind boundary, $h(t)$ is known, $T_W(t)$ and $T_\infty(t)$ are the temperature of interface and fluid.

To solve Eq. (2), the initial temperature distribution $T(x)$ at t = 0 is known and the boundaries depend on the practical problems. Integrating Eq. (2) from x_i to $x_{(i+1)}$ and from t^m to $t^{(m+1)}$, the following expression is obtained:

$$\begin{cases}\int_{x_i}^{x_{(i+1)}}\int_{t^m}^{t^{(m+1)}}\rho(x,T)c(x,T)\frac{\partial T}{\partial t}dtdx=\int_{x_i}^{x_{(i+1)}}\int_{t^m}^{t^{(m+1)}}\frac{\partial V}{\partial x}dtdx\\ \quad+\int_{x_i}^{x_{(i+1)}}\int_{t^m}^{t^{(m+1)}}s(x,t)dtdx \qquad (i=1\sim n)\\ \int_{x_i}^{x_{(i+1)}}\frac{\partial T}{\partial x}dx=\int_{x_i}^{x_{(i+1)}}\frac{V}{k(x,T)}dx \qquad (t=0\sim t)\end{cases} \tag{7}$$

The one-dimensional model is discretized as shown in Fig. 1. The derivatives can be simplified by the difference between two different nodes as $\frac{\partial T_i}{\partial x}=\frac{T_{i+1}^m-T_i^m}{\Delta x}$ and $\frac{\partial V_i}{\partial x}=\frac{V_{i+1}^m-V_i^m}{\Delta x}$. The time derivative is also simplified by the difference between two different moments as $\frac{\partial T_i}{\partial t}=\frac{T_i^{(m+1)}-T_i^m}{\Delta t}$, where $T_i^{(m+1)}$ indicates the temperature at x_i and $t^{(m+1)}$ moment (next time step) while T_i^m is the temperature at current time step. In Eq. (7), V and T are functions of time. A weighted interpolation of variables at two adjacent time moments is adopted to approximate the average values as: $\overline{T}=\theta T^{(m+1)}+(1-\theta)T^m$, $\overline{V}=\theta V^{(m+1)}+(1-\theta)V^m$, where $\theta\in[0,1]$. Therefore, the integral of T and V over time can be written as:

$$\int_{t_m}^{t_{(m+1)}}Vdt=[\theta V^{(m+1)}+(1-\theta)V^m]\Delta t^m \tag{8}$$

$$\int_{t_m}^{t_{(m+1)}}Tdt=[\theta T^{(m+1)}+(1-\theta)T^m]\Delta t^m \tag{9}$$

The different values of θ account for the different weights of $t^{(m+1)}$ and t^m moments. When $\theta=0$, the average variables only depend on current moment t^m which is called pure explicit Euler method. While for $\theta=1$, the average variables equal to the values at next time moment $t^{(m+1)}$ which is called pure implicit Euler method. While for $\theta=1/2$, it is the Crank-Nicolson method