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Solution of inverse radiation-conduction problems using a Kalman filter coupled with the recursive least–square estimator

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ABSTRACT

This paper presents a non–intrusive inverse heat transfer procedure for predicting the time–varying heat flux on the surface of semitransparent media. Inverse radiation–conduction problems were analyzed using a Kalman filter coupled with a recursive least–square estimator (KF–RLSE). Since its unique capability to make fast predictions, KF–RLSE can be easily integrated to existing real–time control systems of industrial facilities. The performance of KF–RLSE was examined thoroughly in a series of numerical simulations in two semitransparent materials (i.e. the glass with black coating and the ceramic of zirconium dioxide ZrO_2) to extract the time–varying surface heat flux on–line from the measured temperature history at boundary. Results showed that the proposed method can predict the unknown boundary flux with an acceptable error. The influence of different parameters on the accuracy and stability of the predicted heat flux was also investigated. Results indicated that the sensor location, process noise covariance and absorption coefficient exerted stronger effects on retrieval results compared with other parameters.

1. Introduction

In the last few decades, on–line tracking of the time–dependent boundary heat flux or other physical parameters has attracted significant attention because of its wide applications in many industrial and engineering fields, such as in the time–varying heat flux measurement of high–speed continuous emission of barrel tubes or highly–integrated electronic products, the internal temperature monitoring of engine combustion chambers, the flame temperature detection in coal–fired furnaces, the internal temperature variation measurement of tissues irradiated by lasers, and estimating the time–varying phase thickness and shape of banks in high temperature heating furnace, to name a few [1–7]. With the need to estimate the history of unknown properties in real time, the recursive input estimation algorithm of digital estimation theory based on the Kalman filter (KF) technique and the recursive least–squares estimation (RLSE) was developed [8]. In theory, the KF technique coupled with RLSE is by virtue of recursive algorithm; thus, an on–line estimation can be employed in place of off–line estimation. It is computationally efficient because it has a simple mathematical formulation and only needs the measurement information at current moment. In previous years, sustained efforts have been aimed at estimating the boundary heat flux by using filtering–based methods owing to their remarkable characteristics. As an efficient method for various real–time controls, filtering technology was first introduced by Beck [3–5] to solve inverse heat conduction problems (IHCPs). Filtering technology is not only a new method of solving IHCPs, but also a representation of the solution in a form suitable for continuous (on–line) processing of data. Ji et al. [9] applied KF coupled with RLSE (KF–RLSE) to retrieve the boundary heat flux of the transient heat transfer on a one–dimensional (1D) slab successfully. Since then, KF has drawn much attention and numerous studies have focused predicting the time–varying heat flux by using KF. Ijaz et al. [10] employed KF to solve a two–dimensional (2D) transient IHCPs. Similarly, Chen and Liu [11] employed a combination of KF and RLSE to determine heat flux history on a rocket nozzle liner. Based on the basic KF method, a variety of improved KF methods were developed to solve the IHCPs. For instance, Daouas and Radhouani [12] have proposed an extended KF with augmented state to predict the input heat flux imposed at the boundary of stainless steel. Chen et al. [13] utilized the extended KF coupled weighted recursive least square method to retrieve the time–varying boundary heat flux of nonlinear heat conduction problems with high precision. Jang and Cheng et al. [14] combined the extended KF with RLSE to predict the on–line heat dissipation of an electronic device. Moreover, Wang et al. [15] applied the extended KF coupled with the RLSE to predict the heat flux imposed on a 1D slab. Aside from determining the

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Nomenclature

$\mathbf{A}$	coefficient matrix
B	sensitivity coefficient
$\mathbf{C}$	coefficient matrix
c_p	the specific heat, J/kg K
E	total element number
$\mathbf{F}$	coefficient matrix
G	the incident radiation, W/m^2
h	the convectional heat transfer coefficient, W/m^2 K
$\mathbf{H}$	measurement matrix
I	the radiative intensity, W/(m^2 sr)
$\mathbf{I}$	identity matrix
k	time (discretized), s
$\mathbf{K}$	the Kalman gain
$\mathbf{K}_b$	the steady–state correction gain
L	the medium thickness, m
$\mathbf{M}$	the sensitivity matrix
n	the refractive index of the media
$\mathbf{N}$	coefficient matrix
$\mathbf{P}$	the filter's error covariance matrix
$\mathbf{P}_b$	the error covariance matrix
q	the heat flux, W/m^2
$\hat{q}$	the estimated input vector, W/m^2
q_r	the radiation heat flux term, W/m^2
Q	process noise covariance
R	measurement noise covariance
s	innovation covariance
t	time, s
T	temperature, K
T_S	ambient temperature, K
T_0	initial temperature, K
$\mathbf{U}$	coefficient matrix
w	process noise vector
x	the x–axis coordinate
$\mathbf{X}$	the state vector
$\bar{\mathbf{X}}$	the input estimator
Z	the observation vector
$\bar{Z}$	the bias innovation

Greek symbols

β	the extinction coefficient, m^{-1}
γ	the forgetting factor
ε	the boundary emissivity
κ_a	the absorption coefficient, m^{-1}
λ	conductivity, W/(m K)
υ	measurement noise vector
ρ	density, kg/m^3
σ	the Stefan–Boltzmann constant
σ_s	the scattering coefficient, m^{-1}
σ_R	standard deviation
$\mathbf{\Phi}$	state transition matrix
$\Phi(\mathbf{\Omega}', \mathbf{\Omega})$	scattering phase function
$\mathbf{\Omega}$	the solid angle, sr
ω	scattering albedo or the inertia weight factor
Γ	input matrix

Superscripts

0	the last moment
k	time, s
T	transpose of a matrix

Subscripts

b	blackbody
E, P, W	the position of node
n	element number
w1	the left boundary
w2	the right boundary

heat flux, considerable attention has also been given to estimation of the time–varying thickness in high temperature phase change procedure by using KF–based techniques. For example, LeBreux et al. [16] applied KF–RLSE to estimate the thickness of a time–varying bank in a high temperature heating furnace for the real–time control of the bank in the optimal thickness. Evidently, an accurate estimation of the time–varying bank thickness is important to improve the production efficiency and avoid safety accidents. They [17,18] also employed three KF models, i.e., a nonlinear unscented KF, a nonlinear extended KF, and a linear KF, to use the inverse heat transfer method in predicting the ledge thickness inside high–temperature metallurgical reactors. As they pointed out, the predictions of the unscented KF are more accurate than those of the linear KF, and more stable than those of the extended KF, and its CPU time requirement is comparable with that of other KF models. Recently, Noh et al. [19] have employed KF to determine heat flux history on a barrel tube. They found that the KF model has higher inversion accuracy when the direct problem is solved using the thermal resistance network method instead of the finite element method, which proves that the inversion accuracy is highly related to the forward model.

In sum, all the studies above have focused on solving the pure conduction or phase change problems in various applications by using KF–based methods. Nowadays, semitransparent media (e.g., thermal insulation ceramics, glass melt, molten salt, and porous media) have been widely utilized in high–tech industrial and scientific fields, especially in ultra–high–speed aircraft and space shuttle [20]. For practical purposes, on–line tracking the time–varying heat flux or temperature distribution in such semitransparent media is urgently needed and has received considerable treatment in recent decades [21]. For example, worthy design of aerospace thermal protection systems (TPS), which is composed of semitransparent materials, largely depends on the ability to accurately predict and understand the transient coupled radiation–conduction heat transfer phenomena that accompany shock–shock interactions and atmospheric re–entry [22,23]. Accurate knowledge of the transient flux profile within a thermal barrier coating (TBC) is critical in evaluating the performance and monitoring the health of a TBC [24,25]. Meanwhile, the primary heat transfer mechanism to the aero optical–thermal effect from a hypersonic vehicle is thermal convection from the high–speed air to the window surface, which exhibits a highly nonlinear behavior [26]. The time–resolved heat flux acting on the surface wall, which is dominated by the coupled radiation–conduction heat transfer in the semitransparent optical window, is of great significance for thermal control or design and difficult to probe manually from the internal wall temperature. Difficulties with probing the time–varying heat flux manually from the internal wall temperature measurement have resulted in the lack of an on–line inverse model for addressing this issue. How to estimate the time–varying temperature of the heating surface is very important for the optimal design of optical window's material and structure, in which the coupled conduction–radiation heat transfer should be taken into account [27–29]. However, to the best of our knowledge,