International Journal of Heat and Mass Transfer 111 (2017) 643–650

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Unsteady mixed convection over an exponentially decreasing external flow velocity

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A R T I C L E I N F O

Article history:
Received 30 November 2016
Received in revised form 5 April 2017
Accepted 5 April 2017

Keywords:
Exponentially decreasing free stream
Heat source/sink
Mixed convection
Non-similar solution
Wall suction/blowing
Unsteady effects

A B S T R A C T

A very recent study of Patil et al. (2017) on the effects of steady mixed convection flow with an exponentially decreasing free stream velocity motivates this research investigation. The present analysis reveals the mixed convection impact over an exponentially decreasing free stream velocity in an unsteady incompressible laminar boundary layer flow involving the effects of suction or blowing and heat generation or absorption. By utilizing the appropriate non-similar transformations, the complex dimensional governing boundary layer equations are simplified into dimensionless equations. In order to prevail the mathematical convolutions in attaining the non-similar solutions at the leading edge of the streamwise coordinate as well as non-similarity variable ξ, the coalition of implicit finite difference scheme and the Quasi-linearization technique is used with the suitable step sizes along the streamwise and time directions. The effects of various dimensionless physical parameters over the momentum and thermal fields are examined. The range of the parameters studied in this analysis are taken as $\alpha(-0.5 \leqslant \alpha \leqslant 1.0)$, $\xi(0 \leqslant \xi \leqslant 1)$, $\tau(0 \leqslant \tau \leqslant 1)$, $\varepsilon(0.000001 \leqslant \varepsilon \leqslant 0.1)$, $Ri(-3 \leqslant Ri \leqslant 10)$, $Re(10 \leqslant Re \leqslant 250)$, $A(-1 \leqslant A \leqslant 1)$ and $Q(-1 \leqslant Q \leqslant 1)$. Further, the present numerical investigation is focussed towards unsteady flow and heat transfer characteristics.

1. Introduction

In a day today life, each and everything is connected with the time. Especially in the fluid flow, the variation of flow variables is all way around associated with the time. Hence the concept of unsteady [2–4] impinge its importance in every aspects of the fluid flow. Unsteadiness may occur due to the body motion which rely on the time variable or it may be due to surrounding field disruptions. That is, the variations due to suction/injection or the body motion by velocity disturbances or the free stream velocity or heat source/sink or all these variations at the same time may cause the unsteadiness in the fluid flow. The stall flutter of helicopter rotor blades [5], dynamic stall of lifts surfaces [6] and the flow through turbo machinery blades [5] are some of the areas of implementation of unsteady motion. Since last decade, many researchers are working on unsteady flows and most of them have faced difficulties because of the significant hardness while solving the problems of unsteady flows. Also, the idea of non-similarity in the fluid flow offers plenty of hurdles to the researcher. The curvature of the body or the free stream velocity or the suction/injection at the surface or all these facts may yield the non-similarity in the fluid flow. Patil et al. [1] have already considered the study on steady flow over an exponentially decreasing free stream velocity and have discussed the effects of suction/injection and heat source/sink over the velocity flow. So, as a challenging task, in this research paper, we are considering the unsteady mixed convection flow over an exponentially decreasing free stream velocity and will discuss the unsteadiness over the flow and also the effects due to the unsteady flows.

Curle [7] has perused the steady two-dimensional laminar incompressible boundary layer by considering the external flow velocity as $u_e = u_0(1 - \varepsilon e^{\xi})$, $0 < \varepsilon < 1$, where u_o is constant, ε is a small parameter and ξ is a scaled streamwise co-ordinate. The smaller values of ε and ξ results in the weaker effects of εe^{ξ} and which clearly approximates u_e as a constant. However, the increase in the values of ξ increases the effect of εe^{ξ} and as ξ approaches to $\log(\varepsilon^{-1})$, u_e suddenly declines and causes the boundary layer

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.04.016
0017-9310/

Nomenclature

A	suction/blowing parameter
C_f	local skin-friction coefficient
C_p	specific heat at constant pressure
f	dimensionless stream function
F	dimensionless velocity
G	dimensionless temperature
g	acceleration due to gravity
Gr	Grashof number due to temperature
L	characteristic length
Nu	Nusselt number
Pr	Prandtl number
Q	heat generation/absorption parameter
Re	Reynolds number
Ri	Richardson number
T	temperature
T_w	temperature at the wall
T_{w_0}	reference temperature
T_∞	ambient temperature of the fluid
t	time
u, v	velocity components in x and y directions respectively
U_0	reference velocity
U_w	wall velocity
U_∞	free stream velocity constant
x, y	Cartesian coordinates

Greek symbols

α	unsteady parameter
α_m	thermal diffusivity
β	volumetric coefficient of the thermal expansion
ε	decelerated parameter
η	similarity variable
$\phi(\tau)$	unsteady function of τ
ξ	non-similarity variable
ξ, η, τ	transformed variables
μ	dynamic viscosity
v	kinematic viscosity
ρ	density
ψ	stream function

Subscripts

0	value at the wall for $\tau = 0$
e	free stream condition
i	initial condition
ξ, η, τ	denote the partial derivatives with respect to these variables, respectively
t, x, y	denote the partial derivatives with respect to these variables, respectively
w, ∞	conditions at the wall and infinity, respectively

separation. That is, the smaller value of u_e implies the adverse pressure gradient. In other words, the thickness of the thin boundary layer grows in presence of adverse pressure gradient and ultimately the boundary layer breaks away from the bounding wall. The loss of lift [8], diminished pressure recovery [8] and increase in the drag [8], etc. are all due to the separation of boundary layer. So, it is important to control the boundary layer separation and many researchers are using different methods like imposition of favourable pressure gradient, introducing a transverse magnetic field, suction at the wall and increasing the boundary layer velocity, etc. to control the separation of the boundary layer.

In order to control the boundary layer separation, suction/blowing through a wall slot [9,10] has become one of the powerful tool. Skin-friction reduction on control surfaces [11], thermal protection [12], and energizing of the inner portion of boundary layer in adverse pressure gradient [13] are some of the applications of slot suction/injection. By utilizing the suction/blowing technique, the boundary layer separation can be controlled or the separation can be delayed by some extent. That is, along the surface, suction/blowing influences the growth of boundary layer, which helps to delay/control the separation of the boundary layer. Furthermore, the process of controlling the boundary layer separation can also be accomplished by the technique of heat source/sink [14–16]. That is, in undersea applications, heating enhances the stability by balancing the thermal and momentum boundary layers near the wall. Hence, the combine effects of suction/injection and heat source/sink can be an effective tool in order to overcome the boundary layer separation problem.

Thus, the present work explores the effects of suction/blowing and heat source/sink over the unsteady mixed convection flow of a laminar two-dimensional incompressible fluid over an exponentially decreasing external flow velocity. To our best of knowledge, the results are new and original as this paper is the first to consider such governing equations. Moreover, the Quasi-linearization technique in coalition of finite difference scheme [17–20] is used to solve the coupled non-linear partial differential equations. The numerical results are compared with the results of Patil et al. [1] and Chiam [21] and found that the results agree in excellent fashion. Further, the present work explores the non-similar solutions that have been obtained starting from the origin of the stream-wise coordinate to the point of separation (zero skin-friction in the stream-wise direction) using Quasi-linearization technique in coalition with implicit finite difference scheme. This research paper may be useful in the assimilation of many boundary layer problems that have practical importance, for example, in controlling transition and/or delaying the boundary layer separation over control surfaces and in suppressing the recirculating bubbles.

2. Mathematical analysis

Along the vertical surface, when the mass transfer $v_w(x, t)$ (suction/blowing) occurs, the unsteady mixed-convection boundary layer flow is considered on an exponentially decreasing free-stream velocity distribution. It is taken care that at the edge of the boundary layer, the inviscid flow is not affected by assuming the blowing rate of the fluid as small and also it is assumed that the injected fluid preserves the same physical properties as the boundary layer fluid [22]. Fig. 1 displays the physical model and coordinate system, in which the coordinate x is measured vertically upwards such that $x = 0$ corresponds to the leading edge and the normal coordinate y is measured from the vertical surface. The variations in the density of the fluid flow results in a body force term in momentum equation and other all thermo-physical properties are supposed to be the constants. In order to conjoin the temperature field $[g\beta(T - T_\infty)]$

to flow model, the changes in the density are related by supplicating the Boussinesq approximation for the energy [22]. With all these pre-propound assumptions, the continuity, momentum and energy equations governing the unsteady mixed convection boundary layer flow are [21]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$