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# Convection in a porous medium with variable internal heat source and variable gravity

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ABSTRACT

The qualitative effect of variable internal heat source and variable gravity on the onset of convection in a horizontal fluid saturated sparsely packed porous layer is investigated using linear stability analysis. The single term Galerkin technique is used to compute the value of the critical Rayleigh number and the corresponding wave number. Three different sets of variable gravity and heat source functions are chosen and their influence on the onset of convection is discussed. The eigenvalues are obtained for free-free velocity boundary conditions with isothermal temperature conditions on the boundaries. The influence of porous parameter on the onset of convection is brought out. It is found that the variable heat source and variable gravity does not affect the shape and size of the convective cell at the onset of convection. However the porous parameter do so.

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## 1. Introduction

Thermal convection in a fluid saturated porous medium has attracted the interest of engineers and scientists for a long time due to its numerous applications in fields such as geothermal energy utilization, oil reservoir modeling, building thermal insulation and nuclear waste disposals, to mention a few. The convective instability of horizontal porous layer subjected to a destabilizing temperature gradient has been investigated extensively by several authors, using a Darcy model. Most of the findings relevant to this problem have also been compiled in the works of Combarnous [1] and Bejan [2]. However, now it is understood that the Darcy model is applicable only under special circumstances and a generalized model for the accurate prediction of convection in a porous media must include Forchheimer's inertia term and Brinkman's viscous term. Rudraiah et al. [3] have used the Darcy-Brinkman model for studying Beard convection in a porous media (see Nield and Bejan [15]) .

There are many works that appeared in the literature, concerning how a time periodic boundary temperature affects the onset of Rayleigh-Bénard convection. Most of the findings relevant to these problems have been reviewed by Davis [4]. On the other hand, the studies related to the effect of thermal modulation on the onset of convection in a fluid saturated porous medium have received marginal attention. The effect of time dependent wall temperature on the onset of convection in a porous medium has been studied by Rudraiah and Malashetty [5]. There has been a great deal of interest in the study of the effect of complex body forces on convection in a fluid and fluid saturated porous layer [6–7,13,15–17]. Joseph and Shir [8] studied the effect of internal heating on convection and Joseph [9] has used nonlinear energy method to find the critical Rayleigh number for a fluid saturated porous layer subject to an internal heat source.

The effect of variable gravity and variable internal heat source on the onset of convection in a fluid and fluid saturated porous layer has been studied Straughan [10] and Rionero and Straughan [11] using energy method. The object of this investigation is to study the effect of variable gravity and variable internal heat source on the onset of convection in a sparsely packed porous medium.

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**Nomenclature**

| | |
|---|---|
| $a_c$ | critical wave number |
| $C$ | critical wave number |
| $d$ | depth of the liquid layer |
| $D$ | $d/dz$ |
| $D/Dt$ | material or substantial derivative $(\partial/\partial t + \vec{q}, \cdot\nabla)$ |
| $\underline{g}$ | acceleration due to gravity |
| $g$ | gravitational acceleration (0, 0, – $g$) |
| $g_0$ | constant |
| $\hat{k}$ | unit vector in the vertical direction |
| $K$ | permeability |
| $k_x, k_y$ | wave number in the $x$ and $y$ directions |
| $p$ | pressure |
| $p_b$ | basic state pressure |
| $Pr$ | $(\mu/\kappa\rho_0)$ Prandtl number |
| $\vec{q}$ | $\equiv (u, v, w)$ velocity vector |
| $\vec{q}'$ | velocity of the perturbed state |
| $q(z)$ | proportional to the integral internal of the heat source |
| $Q(z)$ | internal heat source |
| $R^2$ | $(g_0\alpha\rho_0 Kd^2/\mu\kappa)$ Rayleigh number |
| $R_c$ | critical Rayleigh number |
| $S$ | heat capacity ratio |
| $t$ | time |
| $T$ | liquid temperature |
| $T_b$ | basic state temperature |
| $T_0$ | reference temperature |
| $(x, y, z)$ | Cartesian coordinates with $z$-axis vertically upwards |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal expansion coefficient |
| $\delta$ | small amplitude |
| $\delta_{ik}$ | metric tensor |
| $\Delta T$ | temperature difference |
| $\nabla$ | $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (vector differential operator) |
| $\nabla^2$ | $\frac{\partial^2}{\partial x^2}, + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (three dimensional Laplacian operator) |
| $\nabla_1^2$ | $\frac{\partial^2}{\partial x^2}, + \frac{\partial^2}{\partial y^2}$ (two dimensional Laplacian operator) |
| $\varepsilon$ | small amplitude |
| $\kappa$ | thermal diffusivity |
| $\mu$ | dynamic viscosity |
| $v$ | $(\mu/\rho)$ kinematic viscosity |
| $\rho_b$ | basic state density |
| $\rho_0$ | constant |
| $\sigma$ | $(d/\sqrt{K})$porous parameter |
| $\theta$ | nondimensional temperature |

*Subscripts/superscripts*

| | |
|---|---|
| $b$ | basic state |
| $c$ | critical quantity |
| - | time average |
| * | dimensionless quantities |

## 2. Mathematical formulation

We consider a Boussinesq fluid saturated sparsely packed homogeneous porous medium confined between two infinite horizontal surfaces situated at $z = 0$ and $z = d$ with variable gravitational force **g** and a variable internal heat source $Q$ , as shown in the Fig. 1.

Further we assume that the heat source $Q$ and the gravity **g** depends on the vertical coordinate $z$ . Let $\Delta T$ be the temperature difference between the upper and lower surface with $T_u$ the temperature of the upper plate and $T_l$ that of the lower plate $(T_l > T_u)$. With the assumptions and approximations, which are frequently used for thermal convection in a fluid saturated sparsely packed porous medium the governing equations are taken as

$$\frac{\partial \vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}\mathbf{g}(z) - \frac{\mu}{K\rho_0}\vec{q} + \frac{\mu}{\rho_0}\nabla^2\vec{q}, \quad (1)$$

$$S\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa\nabla^2 T + Q(z), \quad (2)$$

$$\nabla\cdot\vec{q} = 0, \quad (3)$$

$$\rho = \rho_0[1 - \alpha(T - T_0)], \quad (4)$$

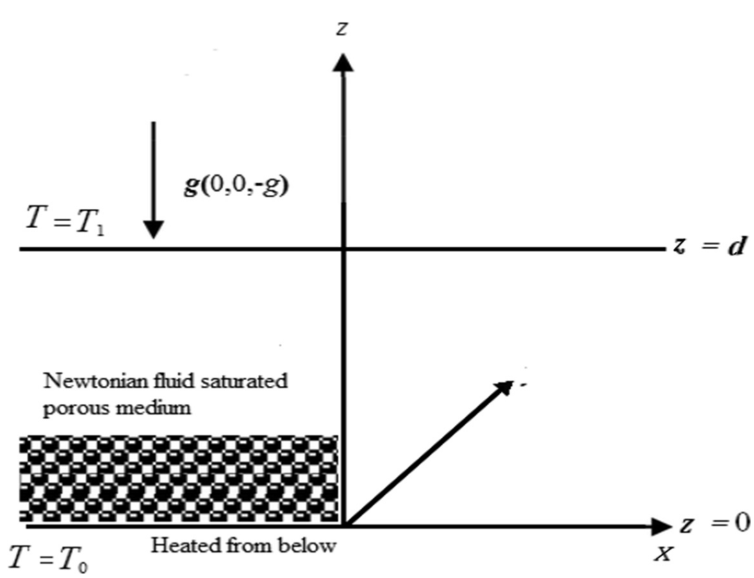


**Fig. 1.** A schematic diagram of the problem.

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