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Peristaltic flow of an Eyring Prandtl fluid in a diverging tube with heat and mass transfer

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ABSTRACT

The present article reflects the peristaltic motion of an Eyring Prandtl fluid in a diverging tube in the presence of heat and mass transfer. Mathematical Analysis has been carried out by considering the assumptions of long wave length and low Reynolds number. Under these limitations, we obtain resulted non linear system of small values. These have been evaluated analytically using homotopy Perturbation method. The succeeding complex disparity computations are explained for velocity, temperature, concentration field and pressure gradient. The effects of physical parameters on the flow and heat transfer attributes are investigated in detail. Special attention is given to explain the pumping and trapping phenomena in detail. Convergence of solution is clearly shown in the graphical results. It is found that rate of heat transfer and mass transfer decreases with increasing Brashof number. The consequence of many underdeveloped restraint are explicate for five dissimilar impressions. Streamlines in trust at the terminus point of the objective.

1. Introduction

Peristaltic is defined as a wave of relaxation contraction imparted to the walls of a flexible conduit, there by pumping the enclosed material. The need for peristaltic pumping may arise in circumstances where it is desirable to avoid using any internal moving parts such as pistons in a pumping process. Moreover, the peristaltic is also a well known mechanism of fluid transport in biological system. Especially it has been found to be involved in swallowing food through the esophagus, urine transport from the kidney to the bladder through the urethra, movement of chime, transport of lymph in the lymphatic vessels and in the vasomotion of small blood vessels such as arteries, venules and capillaries. Roller and finger pumps also operate on this principle. Moreover, peristalsis has been proposed as a mechanism for the transport of spermatozoa in vas deferens (a duct which connects the ducts epididymidis to an ampulla).

Since the first study of peristaltic flow by Latham [1] and Shapiro et al. [2]. Are the first who analyzed the fluid motion in peristaltic pump subject to long wavelength and low Reynolds number approximations. Later on an exhaustive investigations have been made for the peristaltic flow problem with Newtonian and non-Newtonian fluid with different geometries and it can be seen in Kh Mekheimer and Abd Elm [3–5] abound where they discussed the couple stress fluid, influence of heat transfer and magnetic field on a Newtonian fluid in an annulus, Second order non linear fluid through porous medium studied by many researchers [6–12]. In Tripathi and Pandey Das [13] have focused their attention on the study of nano fluid. Peristaltic flow in a tube is discussed by Barton and Raynor [14]. A mathematical model based on viscoelastic fluid flow for the peristaltic flow of chyme in the small intestine has been considered by Tripathi [15]. Occurrence of peristalsis in different parts of the human body has been studied in recent past. Peristalsis in male reproductive system was observed experimentally and numerically by Batra [16], Guha et al. [17], Gupta and Seshadri [18], Srivastava and Srivastava [19]. They modeled the peristaltic flow in the vas deferens by assuming it to be a non-uniform diverging channel and a tube. They looked at a more realistic model by evaluating non-Newtonian (power law fluid) fluid flow in a non-uniform tube. Li and Brasseur [20], Misra and Pandey [21,22] studied both experimentally and analytically the movement of food bolus in the esophagus and gastrointestinal tract. It is also acknowledged that the mechanism of heat transfer is concerned with only two things: temperature and flow of heat, Temperature express the amount of thermal energy from one place to another. Chemical process of heat transfer can be categorized in the following: conduction, convection and radiation. Moreover the importance of thermal

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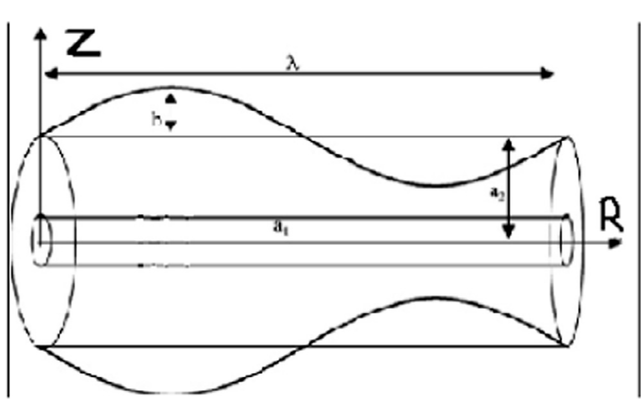
effects of blood in the process like oxygenation and hemodialysis makes the study about the performance of heat transfer in peristalsis very important. Keeping all these viewpoints few recent Refs. [23–27] has considered heat transfer analysis for peristalsis of both Newtonian and non-Newtonian fluid. Many studies can be found with non-Newtonian fluid in a diverging tube such as Biomathematical study of non-Newtonian nanofluid in a diverging tube is discussed by Akbar and Nadeem [28]. Another study of nano Prandtl fluid model in a diverging tube is discussed by Akbar [29]. And Application of Eyring-Powell fluid model in peristalsis with nano particles is also considered by Akbar [30]. Many other studies on peristaltic flow are carbon nanotubes analysis for the peristaltic flow in curved channel with heat transfer [31], entropy generation analysis for a CNT suspension Nanofluid in Plumb Ducts with peristalsis entropy [32].

The present attempt deals with the mathematical analysis of the peristaltic motion of an Eyring Prandtl fluid in a diverging tube in the presence of heat and mass transfer. The primary equations of the Erying Prandtl fluid in cylindrical coordinates are modeled. The resulting non-linear mass, momentum, energy and concentration equations are simplified using low Reynolds number and long wavelength approximations. The resulting non-linear problems are solved and the flow fields, temperature and concentration distributions are calculated using analytical technique homotopy Perturbation technique and are presented graphically. However, numerical integration is performed in order to obtain the graphical depiction of the frictional force and pressure rise. In addition to a sinusoidal wave, the wall description during peristaltic motion is assumed to have the waves: multi-sinusoidal, trapezoidal, triangular and square to analyze their graphical behavior for various flow characteristics.

2. Development of the problem

We have considered the peristaltic flow of an incompressible.

Erying Prandtl fluid in a diverging tube. The cylindrical coordinate system (R', Z') is considered such that Z' -axis is placed along the center of the tube and R' is perpendicular to it. Heat as well as mass transfer phenomenon is taken into account by taking temperature T' and concentration C' at the tube wall. The peristaltic motion is induced by a sinusoidal wave of small amplitude travelling along the tube wall. The mathematical description for the geometry of the problem is stated as follows:



$$h'(Z', t') = a' + b' \sin\left[\left(\frac{2\pi}{\lambda}\right)(Z' - c't')\right],$$

where $a' = a'(Z') = a'_0 + KZ'$ is the radius of the tube at axial distance from the inlet, a'_0 is the radius of the inlet, K is a constant depend on the length of the tube, b' is the wave amplitude, λ is the wavelength c' is the propagating velocity and t' is the time.

3. Physical sketch of the problem

From [33], the fundamental equations of continuity, momentum, energy and concentration are

$$\frac{\partial U'}{\partial R'} + \frac{\partial W'}{\partial Z'} + \frac{U'}{R'} = 0, \tag{1}$$

$$\rho\left[\frac{\partial}{\partial t'} + U'\frac{\partial}{\partial R'} + W'\frac{\partial}{\partial Z'}\right]U' = \left[-\frac{\partial P'}{\partial R'} + \frac{1}{R'}\frac{\partial}{R'}(R'\tau'_{R'R'}) + \frac{\partial}{\partial Z}\tau'_{R'Z'}\right], \tag{2}$$

$$\rho\left[\frac{\partial}{\partial t'} + U'\frac{\partial}{\partial R'} + W'\frac{\partial}{\partial Z'}\right]W' = \left[-\frac{\partial P'}{\partial Z'} + \frac{1}{R'}\frac{\partial}{\partial R'}(R'\tau'_{R'Z'}) + \frac{\partial}{\partial Z'}\tau'_{Z'\bar{Z}}\right], \tag{3}$$

the energy equation is

$$\rho c_p\left[\frac{\partial}{\partial t'} + U'\frac{\partial}{\partial'} + W'\frac{\partial}{\partial Z'}\right]T' = k\left[\frac{\partial^2 T'}{\partial R'^2} + \frac{1}{R'}\frac{\partial T'}{\partial R'} + \frac{\partial^2 T'}{\partial Z'^2}\right] + \left[\tau'_{R'R'}\frac{\partial U'}{\partial R'} + \tau'_{R''Z'}\frac{\partial W'}{\partial R'} + \tau'_{Z''R'}\frac{\partial U'}{\partial Z'} + \tau'_{Z'Z'}\frac{\partial W'}{\partial Z'}\right], \tag{4}$$

and concentration equation is

$$\left[\frac{\partial}{\partial t'} + U'\frac{\partial}{\partial R'} + W'\frac{\partial}{\partial Z'}\right]C' = D\left[\frac{\partial^2 C'}{\partial R'^2} + \frac{1}{R'}\frac{\partial C'}{\partial R'} + \frac{\partial^2 C'}{\partial Z'^2}\right] + \frac{DK_T}{T_m}\left[\frac{\partial^2 T'}{\partial R'^2} + \frac{1}{R'}\frac{\partial'}{\partial R'} + \frac{\partial^2 T'}{\partial Z'^2}\right]. \tag{5}$$

The transformations between the two frames are

$$z' = Z' - ct', \; r' = R' \; w' = W' - C \; u' = U', \tag{6}$$

where u' and w' are the velocity components in the wave frame.

Following non-dimensional quantities are specified to reduce the number of variables in the given equations.

$$\begin{gathered} R = \frac{R'}{a_0}, \; r = \frac{r^r}{a'_0}, \; z = \frac{Z'}{\lambda}, \; u = \frac{\lambda u'}{a_0 c}, \; W = \frac{W'}{c}, \; w = \frac{w'}{c}, \; \varepsilon = \frac{a'_0}{\lambda}, \; \phi = \frac{b'}{a'_0}, \\ U = \frac{\lambda U'}{a_0 c'}, \; p = \frac{a_0^2}{\lambda \mu_0 c}p', \; t = \frac{c't'}{\lambda}, \; \theta = \frac{T' - T_0}{T_1 - T_0}, \; r_1 = \frac{r'_1}{a_2}, \\ h = \frac{h'}{a_0} = 1 + \frac{\lambda kz}{a'_0} + \phi\sin(2\pi z), \quad Re = \frac{\rho c' a'_0}{\mu_0}, \; \delta = \frac{a_2}{\lambda}, \; \theta = \frac{T' - T'_0}{T'_0}, \\ \sigma = \frac{C' - C'_0}{C'_0}, \; E_c = \frac{c'^2}{c_p T'_0}, \; Gr = \frac{g\alpha a_2^3}{\nu^2}(T_1 - T_0), \\ P_r = \frac{\mu_0 c_p}{k}, \; S_c = \frac{\mu}{D\rho}, \; S_r = \frac{\rho D K_T T'_0}{\mu_0 T_m C'_0}, \; \tau = \frac{a_0 \tau'}{c\mu_0}, \; \gamma = \frac{\gamma a'_0}{c'}. \end{gathered} \tag{7}$$

The constitutive equations for the Erying Prandtl fluid model are given by

$$\tau = \frac{A\sinh^{-1}\left\{\frac{1}{c}\left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial w'}{\partial r'}\right)^2\right]^{1/2}\right\}}{\left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial w'}{\partial r'}\right)^2\right]^{1/2}}\frac{\partial w'}{\partial r'}, \tag{8}$$

in which A and C are material constants of the Eyring Prandtl fluid model in which ε, P_r, R_e, and E_c, represent the wave number, Prandtl number, Reynolds number and Eckert number respectively, $B_r = P_r E_c$ is defined as Brink man number, while S_r and S_c are the Soret and Schmidt number, respectively. Making use of Eqs. (6) and (7) modifies the expressions (1)–(5) into the following forms.