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Asymptotic analysis of heat transfer in composite materials with nonlinear thermal properties

Igor V. Andrianov [a], Heiko Topol [b,*], Vladyslav V. Danishevskyy [c]

[a] Institute of General Mechanics, RWTH Aachen University, Templergraben 64, 52062 Aachen, Germany
[b] Center for Advanced Materials, Qatar University, P.O. Box 2713, Doha, Qatar
[c] School of Computing and Mathematics, Keele University, Staffordshire ST5 5BG, UK

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ABSTRACT

We study heat transfer through a composite with periodic microstructure. The thermal conductivity of the constituents is assumed to be temperature-dependent, and it is modeled as a polynomial in terms of the temperature. The thermal resistance between the constituents is taken to be nonlinear. In order to determine the effective thermal properties of the material, we apply the asymptotic homogenization method. We discuss different approaches to determine these effective properties for the different volume fractions of the inclusions. For high volume fractions of the inclusion, we apply the lubrication theory. In the case of low volume fractions of the inclusions, we apply the three-phase model. Comparing some special cases of our results to existing ones in the literature shows a good accuracy.

1. Introduction

Modeling of the thermal properties of composites might be challenging, especially when the size of the heterogeneities is significantly smaller than the macroscopic size of the considered structure. In order to simplify the treatment of heat diffusion problems, different approaches have been developed, in which the original heterogeneous material is replaced by a homogenized or effective material with the same macroscopic properties as the original heterogeneous material. Early works on this topic are, for example, the works of Hershey [1], Hill [2], Kerner [3], Kröner [4], Keller [5], and van der Poel [6]. Examples for works on computational homogenization are article of Özdemir et al. [7], and the work of Geers et al. [8] discusses some trends and challenges in this field.

A powerful and wide-spread technique denoted as the asymptotic homogenization method (AHM) has been developed in order to obtain the effective properties of different asymptotic orders of heterogeneous materials with periodic microstructures. The theory behind this technique is described, for example, in the books of Bensoussan et al. [9] and Panasenko [10]. The AHM allows to investigate a macroscopic boundary value problem within a single repeated unit cell of the microstructure. In this approach, a small parameter is introduced, which relates the size of the heterogeneities to the size of the macroscopic problem. The original coordinate variables are then replaced by so-called fast coordinate variables, which consider the problem on the micro-scale, and by slow coordinates, which consider the problem on the macro-scale. The AHM has been applied to analyze different types of homogenization problem, for example to investigate wave propagation in fiber-reinforced composites [11,12]. There also exist numerous articles, which have applied the AHM to determine the effective thermal properties of composites, for example, Allaire [13] and Zhang et al. [14]. Telega et al. [15] applied the AHM to study heat transfer, which is formulated as a minimization problem. Gałka et al. [16] took temperature-dependent thermal parameters of the constituents in the homogenization procedure into account. Allaire and Habibi [17] and Yang et al. [18] analyze heat transfer in porous materials, and they include conduction, convection, and radiation into their considerations. A popular method to investigate the effective properties of composites with a low volume fraction of the inclusion phase is denoted as the three-phase model [19]. The application of such model for the AHM has been discussed and justified in [20]. If the volume fraction of the inclusions approach its maximum, then the close packing model [21], also denoted as the lubrication theory, has been applied in different works. A broader review of trends of the application of the AHM to obtain the effective properties of composites is provided by Kalamkarov et al. [22], who state that the different developed

* Corresponding author.
E-mail address: htopol@qu.edu.qa (H. Topol).

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methods reveal their strengths and disadvantages, and therefore these methods have to be treated as complementary tools.

The effective macroscopic properties result from the properties and the distribution of the constituents, but also from the interaction of the constituents. Composites might reveal thermal resistance between the different constituents, which might for example result from imperfect contact, cracks, or from an interphase material. An early work on thermal interfacial resistance is the article of Kapitza [23]. Examples for composites with coated inclusions is micro-encapsulated paraffin-spheres, which has been studied in different experiments on thermal regulations of buildings [24,25]. Theoretical modeling works on the effective thermal properties, which consider such resistance, are, for example, Quang et al. [26–28] and Andrianov et al. [29]. There exist different interface models which have been taken into account in different studies, such as hybrid interphase regions [30], and inhomogeneous interphases [31].

Our article is organized as follows: In Section 2 we introduce the herein considered boundary value problem, the applied heat diffusion model, and thermal resistance models. In Section 3 we discuss the application of the AHM in order to obtain the effective thermal parameters of the considered composite. The case of large volume fractions of the inclusion is discussed in detail in Section 4, as well as the case of a layered composite. Illustrative examples are introduced to discuss the different features of the derived heat propagation models. In Section 5 we apply the three-phase model for composites with low volume fractions of the inclusions, and we discuss the cases of parallel fibers and spherical inclusions in the matrix. Special cases of our results are compared to known results from the literature. In the final section, we discuss the obtained results, and we provide a brief outlook.

2. Nonlinear heat diffusion in a composite

Consider a heterogeneous material with a periodic microstructure, which is assumed to consist of two constituents, the inclusion $\Omega^{(1)}$ and the surrounding matrix $\Omega^{(2)}$. In the framework of this article we will mainly focus on inclusions of spherical shape, as shown in Fig. 1, and on inclusions of cylindrical shape. In a Cartesian coordinate system with the three base unit vectors $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$, the microstructure of the material can be described by repeated unit cells in form of parallelepipeds of the lengths ℓ_k in the $\mathbf{E}_k$-directions, $k = 1, 2, 3$. The volume of such unit cell then becomes $v = \ell_1 \ell_2 \ell_3$. In the following, we want to study heat diffusion in such composite. Section 2.1 gives a brief general summary on the applied heat diffusion model, and Section 2.2 specifies such model for heat diffusion in a composite. The interaction of the constituents has a crucial role in the behavior of the overall thermal properties, and we consider thermal resistance at the common interface $\partial\Omega^{(1,2)}$ of $\Omega^{(1)}$ and $\Omega^{(2)}$.

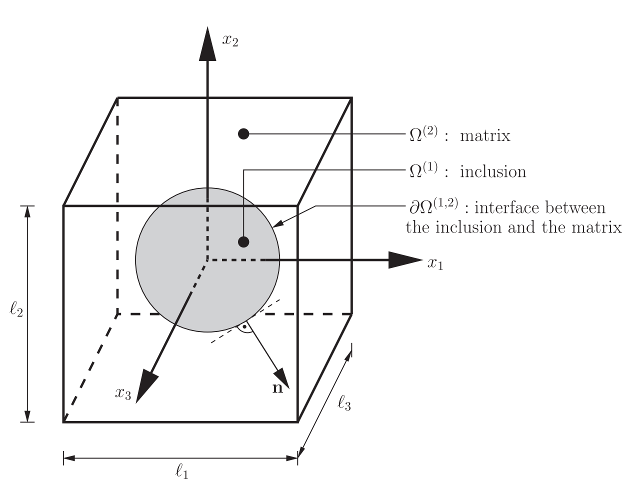

Fig. 1. A single unit cell of the periodic composite microstructure: The inclusion $\Omega^{(1)}$ is surrounded by the matrix $\Omega^{(2)}$. The interface between $\Omega^{(1)}$ and $\Omega^{(2)}$ is denoted as $\partial\Omega^{(1,2)}$, and $\mathbf{n}$ is the outer normal unit vector to the inclusion.

2.1. Summary of the heat equation model

The heat energy flux $q = q(T(\mathbf{x}, t))$ for a material with isotropic thermal properties is given by

$$q(T(\mathbf{x}, t)) = -\kappa(T(\mathbf{x}, t)) \frac{\partial T(\mathbf{x}, t)}{\partial x_k}, \qquad k = 1, 2, 3, \tag{1}$$

where $\kappa = \kappa(T(\mathbf{x}, t))$ is the thermal conductivity and $T = T(\mathbf{x}, t)$ is the temperature at the location

$$\mathbf{x} = \mathbf{E}_1 x_1 + \mathbf{E}_2 x_2 + \mathbf{E}_3 x_3, \tag{2}$$

at time t. Note that (1) represent a form of the heat flux equation in which the thermal properties are taken to be independent from the considered direction. We model the thermal conductivity $\kappa = \kappa(T(\mathbf{x}, t))$ as a function of the temperature, and therefore it is taken to be a polynomial in terms of the temperature in the form

$$\kappa(T(\mathbf{x}, t)) = \sum_{i=0}^{i_{max}} a_i [T(\mathbf{x}, t)]^i = a_0 + a_1 T(\mathbf{x}, t) + \ldots, \tag{3}$$

where a_i are constants. Such model has been applied, for example, by Lienemann et al. [32], and this general form allows to describe different types of effects: The first term of the right side of (3) is the linear term, which is independent from the temperature. The following higher-order terms define the temperature-dependence of the conductivity. The number of terms in (3) depends on the accuracy of the conductivity model in the considered temperature range. If for example the considered temperature range is low, then it might be sufficient to restrict (3) to the leading term. On a large temperature range the thermal conductivity might reveal a strong nonlinear behavior. To give an example, on the temperature range from 0 Kelvin to its melting point, the conductivity of aluminum strongly increases to its maximum, and then slowly decreases with rising temperatures (see, for example, Table 3a in [33]). Thermal conductivities in the form $\kappa = a_m T^m, m = 2, 3, \ldots$ have been studies in different works, and for an overview of the different application of the specific stipulations of this power law forms, we refer to Hristov [34].

In the following we restrict the polynomial expansion of the conductivity to $i_{max} = 1$, so that terms of an order higher than explicitly shown on the right side of (3) will be neglected.

The heat equation which describes the nonlinear heat propagation is taken the form

$$\sum_{k=1}^{3} \frac{\partial}{\partial x_k} \left(\kappa \frac{\partial T}{\partial x_k} \right) = \rho_p \frac{\partial T}{\partial t}, \tag{4}$$

where $\rho_p = \rho_p(\mathbf{x}) = c_p\, \rho$ is the product of the specific heat capacity $c_p = c_p(\mathbf{x})$ and the mass density $\rho = \rho(\mathbf{x})$ of the material. While ρ_p is taken to be independent from the temperature in this article, this shall be noted that this parameter reveals a strong temperature-dependence in the case of phase-changes [25]. After the substitution the specific stipulation of the thermal conductivity (3) for $i_{max} = 1$ into the heat Eq. (4), we obtain a nonlinear heat equation in the form

$$\sum_{k=1}^{3} \left\{ a_0 \frac{\partial^2 T}{\partial x_k^2} + a_1 \left[\left(\frac{\partial T}{\partial x_k} \right)^2 + T \frac{\partial^2 T}{\partial x_k^2} \right] \right\} = \rho_p \frac{\partial T}{\partial t}. \tag{5}$$