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# Bioconvection in rotating system immersed in nanofluid with temperature dependent viscosity and thermal conductivity

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ABSTRACT

This paper investigates the bioconvection in rotating system between two rotating plates immersed in a nanofluid with temperature dependent viscosity and thermal conductivity. The passively controlled model is introduced to characterize the nanoparticle concentration on the upper plate. By means of the similarity transformation, the proposed governing equations are reduced to a class of coupled ODEs with boundary conditions and then the numerical solutions are obtained by the Matlab bvp4c ODE solver. Some important characteristics of velocity, temperature, nanoparticle concentration and density of the motile microorganisms are displayed graphically and discussed in detail. Results show that the viscosity variation parameter has remarkable influence on the local skin friction coefficient and Sherwood number, while local Nusselt number and wall motile microorganisms flux are more sensitive to the thermal conductivity variation parameter. Higher bioconvection Péclet number leads to the aggregation of the motile microorganisms in the middle of the two plates. Moreover, the aggregation of motile microorganisms is weakened by the intense Brownian motion, but improved by the thermophoresis effect.



## 1. Introduction

Bioconvection has been studied extensively due to its wide applications in the modeling oil, gas-bearing sedimentary basin, microbial enhanced oil recovery and bio-microsystems [1,2]. The motile microorganisms are usually heavier than water and they swim upward when stimuli such as light and chemical attraction occur, which arouse the unstable density stratification and hydrodynamic instability [3]. With the development of nanofluids, some researchers brought about an idea of adding motile microorganisms to the nanofluids, which could achieve both enhanced thermal performance and green, sustainable characteristics and simultaneously lay a foundation for the next generation of biofuels. In the near-wall regions of bio-engineered devices, boundary-layer phenomena play vital roles and such fluid mechanics consist of the conservation of momentum, heat transport and mass. Kuznetsov [4] obtained the perturbation solutions to the onset of bioconvection in horizontal layer with nanofluids containing gyrotactic microorganisms. Xu and Pop [5] introduced the passively controlled nanofluid model proposed by Kuznetsov and Nield [6] to model the problem of a fully developed mixed bioconvection flow between two paralleled horizontal flat plates. Uddin et al. [7] applied the improved homotopy analysis technique when considered the influences of the Stefan blowing and the velocity and thermal slips on the nanofluid containing motile microorganisms over a horizontal stretching/shrinking sheet. Raees et al. [8] considered the three-dimensional stagnation flow of a nanofluid containing both nanoparticles and microorganisms on a moving plate with anistropic slip. Based on the study of the MHD non-Newtonian nanofluids over a cone [9], Raju and Sandeep [10] did a comparative research and found that the heat and mass transfer in MHD non-Newtonian bioconvection flow over a rotating cone was significantly high when compared with the flow over a rotating plate. Other researchers have done several work related to the bioconvection, such as the MHD stagnation point flow of gyrotactic microorganisms [11,12], the unsteady mixed bioconvection flow [13,14] and the bioconvection in porous medium [15–17].

Previous studies of the nanofluids bioconvection containing nanoparticles and motile microorganisms were restricted, in general, to the case where the fluid properties were taken as constants. However, the heat generated by the internal friction and the corresponding rise in temperature do affect the viscosity and thermal conductivity of the fluid and those properties can no longer be treated as constants [18]. For example, the viscosity of water at 10 °C ($\mu = 0.0131\ \mathrm{g\,cm^{-1}\,s^{-1}}$) is 2.4 times of that at 50 °C ($\mu = 0.00548\ \mathrm{g\,cm^{-1}\,s^{-1}}$). Hossain and Munir [19] studied the mixed convection flow from a vertical flat plate assuming the

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viscosity of the fluid to be inversely proportional to a linear function of temperature. Attia [20] investigated the unsteady hydromagnetic Couette flow of dust fluid under exponential decaying pressure gradient and Saravanan and Kandaswamy [21] studied the hydromagnetic stability of convective flow with internal heat sources. Those authors treated the viscosity of the fluid as the exponential function of temperature and assumed the thermal conductivity as the linear function of temperature. Umavathi [22,23] considered the free convective flow in vertical rectangular duct filled with porous matrix with temperature dependent viscosity and thermal conductivity. Makinde and Mishra [24] examined the effects of thermal radiation on stagnation point boundary layer flow of variable viscosity water-based nanofluid over a convectively heated stretching surfacing. Effects of the variable thermal conductivity on the viscoelastic nanofluid [25], Sisko nanofluid [26] and CNTS suspended nanofluid [27] were also studied.

Taking the temperature-dependent viscosity and thermal conductivity into account, this paper extends the nanofluid bioconvection problem to the case of three-dimensional rotating system for its potential applications in the micro-electro mechanical system as well as the design in microfluidic devices. Based on the similarity transformation, the governing equations with boundary conditions are transformed into a boundary value problem of ordinary differential equations and the Matlab bvp4c ODE solver is effective to solve it. The effects of the relevant parameters, such as the viscosity parameter, thermal variation parameter, Brownian motion, thermophoresis effect and bioconvection Péclet number on the velocity, temperature, nanoparticle concentration and microorganism density are shown graphically and discussed in detail. Some important physical characteristics on the lower plate are presented in tables.

## 2. Formulation of the problem

Consider the steady bioconvection nanofluid flow between two horizontal parallel plates rotating around the $y$-axis with a constant angular velocity $\Omega$. Both the fluid and the plates rotate together. The Cartesian coordinate is chosen, where $x$-axis is measured horizontally along the plates, $y$-axis is taken perpendicular to the plates and $z$-axis is normal to the $xy$-plane. The two plates are located at $y=0$ and $y=h$. To make the position of the point (0,0,0) unchanged, the lower plate is stretched linearly by two equal and opposite forces. The physical model is shown in Fig.1. It is assumed that the suspended nanoparticles in the fluid are dilute and the nanoparticles do not affect the swimming direction and velocity of the motile microorganisms. In reference of [28,29], the governing Eqs. (1)–(7) under the above assumptions which contain the conservation of the mass, momentum, energy, nanoparticle volume fraction and motile microorganisms are deduced as the followings.

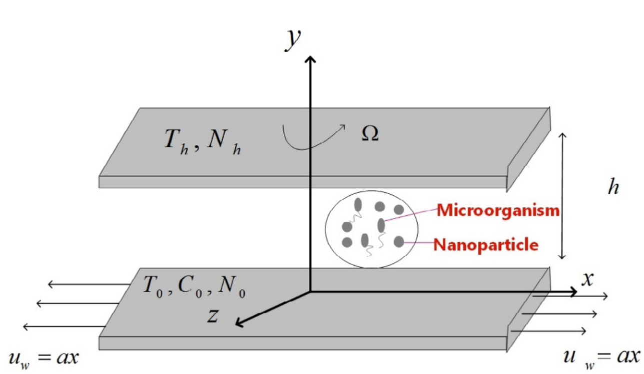


**Fig. 1.** Physical model of the present problem.

$$\nabla \cdot \mathbf{v} = 0. \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + 2\Omega w = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \frac{1}{\rho_f}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \frac{1}{\rho_f}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right), \tag{3}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - 2\Omega u = \frac{1}{\rho_f}\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right), \tag{4}$$

where $\mathbf{v}$ is the velocity vector of the flow with $u$, $v$ and $w$ being the velocity components respectively in x-direction, y-direction and z-direction, $\rho_f$ is the nanofluid density. Fluid viscosity $\mu$ is taken as a function of the temperature in the form of $\mu = \mu_0 e^{-\lambda(T-T_h)/(T_0-T_h)}$ [22,23] where the subscript 0 denotes the reference state ($T = T_0$) and $\lambda$ represents the viscosity variation parameter.

Thermal energy equation:

$$\mathbf{v}\cdot\nabla T = \nabla\cdot\left[\frac{k(T)}{(\rho c_p)_f}\nabla T\right] + \tau\left[D_B\nabla C\cdot\nabla T + \left(\frac{D_T}{T_0}\right)\nabla T\cdot\nabla T\right],$$

where $T$ is the temperature, $C$ is the nanoparticle volume fraction (nanoparticle concentration), $(\rho c_p)_f$ is the heat capacity of the fluid, $\tau = (\rho c_p)_p/(\rho c_p)_f$ is the ratio of the effective heat capacity of the nanoparticle to effective heat capacity to the fluid. Note that the thermal conductivity $k$ is assumed to vary with the temperature following $k = k_0\left[1 + \gamma\frac{(T-T_h)}{(T_0-T_h)}\right]$ [22,23], in which $\gamma$ refers to the thermal conductivity variation parameter. Furthermore, the above equation can be written as follows:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{(\rho c_p)_f}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \left(\frac{D_T}{T_0}\right)\left(\frac{\partial T}{\partial y}\right)^2\right]. \tag{5}$$

Mass transfer equation:

$$\mathbf{v}\cdot\nabla C = D_B\nabla^2 C + \frac{D_T}{T_0}\nabla^2 T,$$

where $D_B$ and $D_T$ are the Brownian motion coefficient and thermophoretic effect coefficient respectively. And we can re-write it as

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_0}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{6}$$

The governing equation of the conservation of microorganisms: $\nabla\cdot\mathbf{j} = 0$.

According to [4], $\mathbf{j}$ is the flux of microorganisms, which is caused due to fluid convection, self-propelled swimming of microorganisms and diffusion of microorganism, and is expressed as

$$\mathbf{j} = N\mathbf{v} + N\tilde{\mathbf{v}} - D_n\nabla N,$$

where $N$ is the density of motile microorganisms, $D_n$ is the diffusivity of microorganism, $\tilde{\mathbf{v}}$ is the average swimming velocity vector of the motile microorganism approximated as $\tilde{\mathbf{v}} = (bW_c/C_0)\nabla C$, in which $b$ is the chemotaxis constant, $W_c$ is the maximum cell swimming speed and $C_0$ is a reference value. Therefore, the governing equation of motile microorganisms is stated as

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + \frac{bW_c}{C_0}\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right) = D_n\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right). \tag{7}$$

The appropriate boundary conditions are

$$\begin{aligned} &u = u_w(x) = ax,\ v = w = 0,\ T = T_0,\ N = N_0,\ C = C_0,\ y = 0,\\ &u = v = w = 0,\ T = T_h,\ N = N_h,\ D_B\tfrac{\partial C}{\partial y} + \tfrac{D_T}{T_0}\tfrac{\partial T}{\partial y} = 0,\ y = h. \end{aligned} \tag{8}$$