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Spectral element simulations of three dimensional convective heat transfer

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ABSTRACT

This paper presents new formulation for simulating three-dimensional convective heat transfer by incorporating the complete viscous dissipation function in the energy equation to account for viscous heating and adopting the Boussinesq approximation for the thermal buoyancy term in the Navier-Stokes equations. In addition, new implementation of fourth order Stiffly Stable Schemes was achieved and tested for time integration. In order to provide dual-level mesh refinement, the *hp*-refinement, for flexible spatial resolutions, a modal spectral element method was used to solve these equations in three dimensions. Simulation results were compared with exact solutions or higher order solutions and good agreement were accomplished. This demonstrates that the new formulation and implementation are accurate enough for investigating convective heat transfer with viscous heating subject to complex thermal and flow boundary conditions in three dimensional irregular domains using the high order version of finite element method.

1. Introduction

Among all the worldwide processes of energy productions and consumptions, about 80% involves heat transfer especially convective heat transfer. Therefore, enhancing the efficiency of heat transfer processes could potentially elevate energy conservation in relevant thermo-fluids engineering [1–6]. Historically, engineering studies of heat transfer processes have been mainly analytically, experimental, empirical or semi-empirical due to technical issues and difficulties [7–10]. In the last three decades with the vast development of computing power, numerical approaches have become one important alternative.

Viscous heating plays an important role in heat transfer especially when the viscosity, shear rate, or temperature gradient is large due to the strong coupling between the energy and momentum equations [11,12]. The heat produced by viscous friction, although small overall, increases the local temperature especially near walls and boundary layers, and decreases the viscosity, and therefore, could dramatically alter local gradients of temperature and velocity [11]. Under certain circumstances, once the rate of heat generation exceeds the rate of heat dissipation into the surrounding environment, the detrimental phenomenon, thermal runaway, also called thermal explosion, could happen [13,14]. Thermal runaway is usually an undesirable process accelerated by the increased temperature which in turn releases more heat and further increases temperature. 3D molecular dynamics simulations of heat and momentum transfer involving viscous heating in nanoscale shear flow were conducted in [15].

Although there are many valuable *analytical studies* on viscous heating, most of them are two-dimensional (2D). Effects of the viscous dissipation were analyzed in 2D with perturbation solution of the governing equation in [16]. The explicit and implicit stabilities of Exact Linear Part schemes for advectiondiffusion equations were studied in [17]. A perturbation method was used to obtain the 2D analytical solution of the momentum and energy equations in [18] and perturbation expansions with respect to a buoyancy parameter were used to analytically evaluate velocity and temperature fields in [19] for convective flows in vertical channels. A 2D study of thermal mechanical effects due to viscous heating in tubes of finite length was completed in [11]. A 2D analytical study on steady magnetohydrodynamics (MHD) flow under convective heating reported more pronounced viscous dissipation effect in nonslip or partial slip flows than slip flows [20]. Another 2D theoretical analysis [21] indicated that viscous dissipation is important for

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flows of relatively large velocities with either the clear fluid model or the Brinkman model.

Handling linear and nonlinear terms separately gains an advantage in numerical heat transfer. Some heat transfer problems are mathematically stiff [22] in the sense that there is a slower process which usually is the heat conduction in thin boundary layers although confined natural convection could be slow as well, and a faster process such as convective mixing. The slower one is mathematically linear and could be handled implicitly with a large time step for fast convergence and improved stability. However, the faster part requires a smaller time step for stability reasons and usually needs an explicit approach to handle the nonlinearity. Therefore, Stiffly Stable Schemes become a good choice in time discretization. Some improvement in linear multistep schemes was reported in [23] where no historical derivative information was required in time integration.

Most *numerical studies* related to viscous dissipation are two-dimensional. The effect of viscous dissipation in a 2D rectangular cavity was investigated with an upwind finite difference scheme along with successive over relaxation in [24]. An essentially 2D finite element solution of a non-Newtonian fluid - polymer melts convective flow in a tube with constant ambient temperature was obtained in [25]. A 2D reciprocating forced convection was simulated in [26] and a 2D thermal convection from uniformly heated walls of a straight channel in presence of a rotationally oscillating cylinder was modeled in [27]. A 2D steady laminar boundary layer flow along a vertical stationary heated plate was simulated using boundary layer equations in [28]. A 2D heat transfer in lid-driven channels with fully developed axial flow for non-Newtonian power-law fluids was studied with the commercial finite element solver *Fastflo* in [29]. A 2D four-square cavity with a uniform heat source and different temperature boundaries was simulated with FLUENT in [8]. To study the heat and mass transfer of MHD micropolar fluids, 2D governing equations were transformed to a set of nonlinear ordinary differential equations with similarity solutions which were then solved numerically by shooting technique in [30]. The viscous heating on forced convection between an unconfined rotating cylinder and a fluid was simulated in 2D settings [12]. An essentially 2D finite difference study was performed on a forced convection in a MHD pump with Joule heating and viscous dissipation in [31]. A 2D finite volume study of wall heating and cooling in a Herschel-Bulkley fluid flow in a circular pipe with uniform wall temperature was conducted in [32]. The 2D governing equations of boundary layer flow and heat transfer of a dusty fluid over an unsteady stretching surface were reduced to nonlinear ordinary differential equations by similarity transformations and then numerically solved with Runge-Kutta-Fehlberg method in [33]. A similar approach was in simulating 2D free convection on a vertical plate in porous media with variable wall temperature in [34].

There is a shortage of numerical study of convective heat transfer using high order methods in both time and space in three dimensional (3D) domains. This paper presents new formulation for convective heat transfer by incorporating the complete 3D viscous dissipation function [35] in the energy equation to account for viscous heating and including the Boussinesq approximation for the thermal buoyancy term in the Navier-Stokes equations. In addition, new implementation of fourth order Stiffly Stable Schemes was achieved and tested for time integration. The dual-level mesh refinement, the *hp*-refinement for varied spatial resolution, was accomplished by using a modal spectral element method [36–41] which solves coupled momentum and energy equations in 3D.

The structure of this paper is outlined as below. Section 1 reviews the current state of research and existing work. Section 2 delineates details of the computational formulation and specific implementations in this paper. Section 3 validates the new formulation by comparing computational results with analytical solutions and presents in-depth discussions. Section 4 describes the application and validation with experimental data. Section 5 summarizes and draws some conclusions.

2. Mathematical descriptions of simulations

2.1. Governing equations

In terms of the primitive variables, $\mathbf{u}, p, T$, the velocity field $\mathbf{u} = (u, v, w)^T$, pressure and temperature, the governing equations consist of balances of the mass, momentum and energy. For an incompressible fluid, the mass conservation equation is:

$$\nabla \cdot \mathbf{u} = 0. \tag{1}$$

For constant properties with up to medium temperature variations, the momentum conservation is described with the incompressible Navier-Stokes equations, in dimensionless units, which are given in the Eulerian form:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla \cdot \nabla \mathbf{u} + \frac{Gr}{Re^2} T \mathbf{z}, \tag{2}$$

where Re and Gr are reference Reynolds number and Grashof number, respectively; T is the dimensionless excess temperature; and $\mathbf{z}$ is the unit vector in the direction of the gravity. The last term in Eq. (2) is the driving force for the natural convection.

In the absence of radiation and ignoring the pressure work, the dimensionless energy conservation equation in terms of the dimensionless excess temperature T is [42]:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{PrRe} \nabla^2 T + \frac{Ec}{Re} \Phi + \frac{\dot{Q} L}{\rho C_p T_0 u_0}, \tag{3}$$

where Pr and Ec are the Prandtl number and Eckert number, respectively; $\dot{Q}$ is the volumetric internal heat source, L, ρ, C_p, T_0, u_0 are the reference values for the length, density, specific heat at fixed pressure, temperature, and velocity, respectively; and Φ is the dimensionless viscous dissipation function as shown below:

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2. \tag{4}$$

A closed system of five scalar equations in Eqs. (1)–(3) in terms of five variables, u, v, w, p and T are to be solved numerically.

2.2. Computational algorithms and implementation

The continuity equation, Eq. (1), is a constraint for the auxiliary variable - pressure in the momentum equations. Assume that four principal variables u, v, w, and T are discretized in space, by the principle of the method of lines [43], the time derivatives are discretized with high order finite difference method, as in the below discussions.

2.2.1. Temporal discretization

Although Adams-Bashforth and Adams-Moulton schemes could be the first choice for the Navier-Stokes equations, to achieve the third order accuracy, the stability condition severely limits the size of time step. The Crank-Nicholson scheme could also be an option for viscous integration. However, using a large time step, a Crank-Nicholson scheme may suffer the short-wave instability [44] and yield unrealistic solutions [45]. Another option is the general θ scheme which provides damping at all frequency for $\theta = 1.0$ and certain stability [46]. Of course for $\theta = 0.5$, it recovers the