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Thermal-solutal capillary-buoyancy flow of a low Prandtl number binary mixture with various capillary ratios in an annular pool

Jia-Jia Yu, You-Rong Li *, Jie-Chao Chen, Yu Zhang, Chun-Mei Wu

Key Laboratory of Low-grade Energy Utilization Technologies and Systems of Ministry of Education, College of Power Engineering, Chongqing University, Chongqing 400044, China

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ABSTRACT

In order to understand the influence of the capillary ratio on the coupled thermal-solutal capillary-buoyancy flow in an annular pool subjected to simultaneous radial temperature and solute concentration gradients, a series of three-dimensional numerical simulations are carried out by using the finite volume method. The annular pool was filled with the silicon-germanium melt with an initial silicon mass fraction of 1.99%. The Prandtl number and the Lewis number of the silicon-germanium melt are 6.37×10^{-3} and 2197.8, respectively. Results indicate that the coupled thermal-solutal capillary-buoyancy flow is steady and axisymmetric when the thermal capillary Reynolds number is relatively small. With the decrease of the capillary ratio, the stable flow pattern experiences three stages, including the single counter-clockwise vortex, the combination of clockwise and counter-clockwise vortexes, and the single clockwise vortex. Besides the special case of the capillary ratio $R_\sigma = -1$, the critical thermal capillary Reynolds number for the incipience of the three-dimensional flow decreases with the decrease of the capillary ratio. Seven kinds of three-dimensional flow patterns are observed in the annular pool, which are the petal-like pattern, spoke pattern, rosebud-like pattern, hydrosolutal waves, ear-like pattern, target-like pattern and copper coin-like pattern. Actual flow pattern is strongly dependent on the capillary ratio, thermal capillary Reynolds number and the aspect ratio.

1. Introduction

The capillary-buoyancy flow driven by the combination of the surface tension gradient and the buoyancy force is strongly related to the temperature and the solute concentration distributions in fluids [1,2]. This flow is essential to exploration in various natural and industrial processes, such as evaporation [3], oceanography [4], solidification of castings and ingots [5], and crystal growth [6,7]. Over the years, the capillary-buoyancy flow is attracting growing interest and has been investigated widely [8–12]. The critical condition of the flow bifurcation and the flow pattern multiplicity have also been reported when rotation [13–15], vibration [16–19] or magnetic filed [20–23] are applied to control the flow. However, most studies focus on pure fluids in which the capillary-buoyancy flow is driven by the thermal capillary and thermal buoyancy forces, while the thermal-solutal capillary-buoyancy flow in binary mixtures is much more complex than the thermal capillary-buoyancy flow in pure fluids due to the coupled effect of thermal and solutal capillary forces, thermal and solutal buoyancy forces.

Ever since Bergman [24] firstly reported the thermal-solutal capillary flow of binary mixtures at a special case where the opposing thermal and solutal capillary effects are of equal magnitude, the continued interest has been attracted on this special cases [25–29].

Bergman [24] verified that, in the absence of buoyancy force, the thermal-solutal capillary flow may occur in a rectangular cavity with a free surface that is imposed by the horizontal temperature and solute concentration gradients, even when the overall thermal and solutal capillary effects are equal and opposite. This flow is depending on the double-diffusion effect. Chen et al. [25,26] further studied the influences of the aspect ratio, Prandtl number, Lewis number on the flow stabilities, flow patterns, transition routes to turbulent flow and so on in rectangular cavities. Recently, Zhou et al. [27,28] paid attention to the dynamic-free surface at this special case. Surprisingly, little attention has been devoted to the thermal-solutal capillary flow in an annular pool. In fact, the annular pool meets the requirement of a spatial extension in the azimuthal direction for travelling waves to establish themselves [29]. Only Chen et al. [29] explored the flow pattern evolution of the thermal-solutal capillary flow of a moderate Prandtl number binary mixture at this special case in an annular pool. A quiescent conductive state was primarily observed at very

* Corresponding author.
E-mail address: liyourong@cqu.edu.cn (Y.-R. Li).

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Nomenclature

C	mass fraction
Bo_C	solutal dynamic Bond number
Bo_T	thermal dynamic Bond number
d	depth, m
D	mass diffusivity of species, m^2/s
$\mathbf{e_z}$	z-directional unit vector
g	gravitational acceleration, g = 9.8 m/s^2
Gr_T	Grashoff number
Le	Lewis number
m	wave number
Nu	Nusselt number
p	pressure, Pa
P	nondimensional pressure
Pr	Prandtl number
r	radius, m
R	nondimensional radius
Re_C	solutal capillary Reynolds number
Re_T	thermal capillary Reynolds number
R_ρ	buoyancy ratio
R_σ	capillary ratio
Sc	Schmidt number
t	time, s
T	temperature, K
v	velocity, m/s
V	nondimensional velocity
$\boldsymbol{V}$	nondimensional velocity vector
z	axial coordinate, m
Z	nondimensional axial coordinate

Greek symbols

α	thermal diffusivity, m^2/s
β_C	solutal expansion coefficient
β_T	thermal expansion coefficient, 1/K
γ_C	solutal coefficient of surface tension, N/m
γ_T	temperature coefficient of surface tension, N/(K m)
ε	aspect ratio
θ	azimuthal coordinate, rad
ν	kinematic viscosity, m^2/s
μ	dynamic viscosity, kg/(m·s)
ρ	density, kg/m^3
τ	nondimensional time
ψ	nondimensional stream function
Θ	dimensionless temperature
Φ	dimensionless solute concentration

Subscripts

ave	average
c	critical
i	inner
o	outer
r, R	radial
z, Z	axial
θ	azimuthal
0	initial

small thermal capillary Reynolds number. A travelling wave, a combination of the travelling wave and stationary wave and a vibrating spoke pattern orderly appear with the increase of the thermal capillary Reynolds number.

Likewise, investigations on the thermal-solutal capillary flow with various capillary ratios are not as extensive as those at the special case shown above. There have been a few studies on the effect of the capillary ratio on the thermal-solutal capillary flow in rectangular cavities [27,30], but few ones pay attention to the flow in an annular pool. In fact, the small intensity of the solutal capillary flow influences the shape of the dynamic free surface [27], the flow bifurcations [30], heat and mass transfer rates [30] and so on.

In this work, we demonstrate a series of three-dimensional (3D) numerical results on the thermal-solutal capillary buoyancy flow with various capillary ratios in an annular pool. Abbasoglu et al. [31] and Matsui et al. [32] reported that the capillary ratio, which is generated by the segregation during Czochralski crystal growth, ranges from around -2.44 to -0.20. Therefore, the capillary ratio of $-2 \le R_\sigma \le 0.2$ is considered. On the other hand, the silicon-germanium melt with a silicon initial mass fraction of 1.99% is chosen as the working fluid because of its wide application in microelectronic devices and optoelectronic.

2. Model formulation

2.1. Basic assumption and governing equations

An annular pool of inner radius r_i, outer radius r_o and depth d is filled with silicon-germanium melt (C_0 = 1.99%), as shown in Fig. 1. The aspect ratio of the annular pool is defined as $\varepsilon = d/(r_o - r_i)$, and the radius ratio is $\eta = r_i/r_o$. In Fig. 1, constant temperatures and solute concentrations, T_i, C_i and T_o, C_o, are imposed on the inner and outer cylinders, respectively. In addition, $T_i < T_o$, while C_i and C_o are dependent on the sign of the capillary ratio, R_σ.

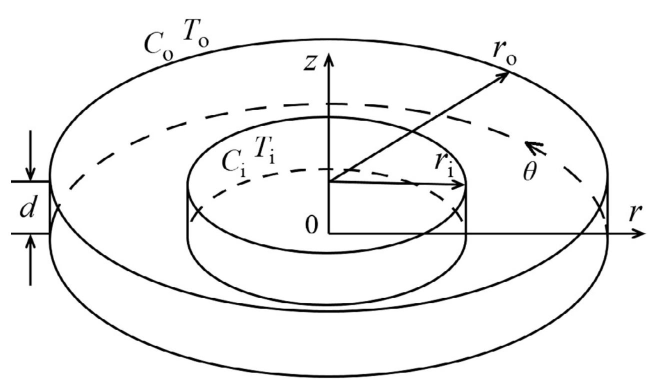

Fig. 1. Physical model and the coordinate system.

In order to simplify the model, it is reasonable to introduce the following assumptions: (1) The bottom is a rigid solid, while the top is a non-deformable and flat free surface. The bottom wall and free surface are assumed as adiabatic and impermeable boundaries. (2) The binary mixture is an incompressible Newtonian fluid whose physical properties are mostly considered as constant. However, the surface tension, σ, and density, ρ, are assumed to be linear functions of temperature and solute concentration in the buoyancy term of the momentum equation. (3) The flow is laminar. (4) The Dufour effect and Soret effect are negligible due to few influences on the thermal-solutal capillary flow of a binary mixture, when the constant temperature and solute concentration differences are imposed on the binary mixture [33,34].