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Nanofluid flow due to rotating disk with variable thickness and homogeneous-heterogeneous reactions

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ABSTRACT

An analysis has been carried out to examine the water based nanofluid flow with silver nanoparticles. Flow due to a rotating disk with variable thickness is considered. Heat and mass transfer subject to volume fraction of nanoparticles and homogeneous-heterogeneous reactions are examined. Homotopy concept is utilized for the development of solutions. Special emphasis is given to the non-dimensional velocity, temperature, concentration, skin friction coefficient and Nusselt numbers. It is observed that with an increase in disk thickness the radial, axial and azimuthal velocities are enhanced. Here surface shear stress rates decays for larger thickness coefficient of disk while increases for volume fraction of silver based nanoparticles. Magnitude of heat transfer rate decreases for Reynold number.

1. Introduction

Low thermal conductivity of fluids is a serious issue for several heat transfer process in the engineering applications. Thus researchers have a looking for an innovative way to improve the thermal conductivity of fluids. Various mechanisms have been suggested for thermal conductivity enlacement of fluids. Out of these suspensions of nanoparticles the base fluid has become quite attractive. Nano-sized metallic particles (copper, silver, gold, titanium or their oxides) are used in traditional heat transfer fluids to form slurries. Nanofluids are dilute suspensions of functionalized solid nanoparticles and potential heat transfer fluids developed with the specific purpose of enhancing the thermal conductivity of fluids. This enhanced feature of nanofluids has led to the plethora of diverse industrial and biomedical applications such as engine cooling, drag reductions refrigeration (domestic refrigerator) chillers, oil engine transference, diesel electric generator as jacket water coolant, boiler exhaust flue gas recovery, cooling of electronics, microwave tubes, high-power lasers, drilling, lubrications, nanofluids in transformer cooling oil, cooling of welding, nuclear systems cooling, heating and cooling of buildings, thermal storage and solar water heating. Word "nanofluid" is credited by Choi and Eastman [1]. Later on Choi et al. [2] used nanoparticles to enhance thermal conductivity of fluids and heat transfer rate. Heat transfer enhancement of nanofluids is investigated by Xuan and Li [3]. Tiwari and Das [4] discussed heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. Effect of heat generation/absorption stagnation point flow of a nano fluid over a surface with convective boundary condition is discussed by Alsaedi et al. [5]. Lakshmi et al. [6] presented effects of diffusion-thermo and thermo-diffusion on two-phase boundary layer flow past a stretching sheet with fluid-particle suspension and chemical reaction. Numerical solutions for magnetohydrodynamic flow of nanofluid over a bidirectional non-linear stretching surface with prescribed surface heat flux boundary is addressed by Mahanthesh et al. [7]. Gireesha et al. [8] elaborated melting heat transfer in boundary layer stagnation-point flow of nanofluid toward a stretching sheet with induced magnetic field.

Theoretical and practical significance of engineering and applied sciences persuade the study of flow in a rotating frame. Dominant geophysical applications include the generating subject relevant with the geophysics towards earth rotation and magma flow in earth's mantle near to crust of the earth. Flow subsists in centrifugal filtration process, food and chemical processing industry, rotating machinery and designs of multi-pore distributor and convertor in a gas–solid fluidized bed are such engineering applications of rotating frames. The problem of rotating disk is discussed by Karman [9]. Erdogan [10] analyzed the fluid flow by non-coaxial rotations of disk and a fluid at infinity. Reasonable literature is now available relating to the flow of Newtonian, viscous and non-Newtonian fluids for rotating disk problems (see [11–17]). Nanofluid flow and rate of heat transfer due to a rotating surface

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is discussed by Turkyilmazoglu [18]. Hayat et al. [19] presented impact of magnetohydrodynamic (MHD) flow of Cu-water nanofluid due to a rotating disk with slip effects. Effects of nonlinear thermal radiation in the rotating flow of ferrofluid over a stretched surface are studied by Mustafa et al. [20]. Mahanthesh et al. [21] worked on mixed convection squeezing three-dimensional flow in a rotating channel filled with nanofluid.

Surfaces of variable thickness have applications in mechanical, architectural, civil, marine and aeronautical engineering. It also assists to decrease the weight of structural elements and refine the utilization of material. However it is seen that very less attention has been paid for the flow due to surfaces with variable thickness. Fang et al. [22] described boundary layer flow over a variable thicked stretched sheet. Subhashini et al. [23] analyzed dual solutions in a thermal diffusive flow past a stretching sheet with variable thickness. Zhang et al. [24] worked on bending collapse of square tubes with variable thickness. Hayat et al. [25] examined flow due to a variable thicked stretching surface with variable thermal conductivity and Cattaneo-Christov heat flux. Ramesh et al. [26] studied stagnation point flow of Casson fluid towards variable stretching sheet with thermal radiation. Laminar flow and heat transfer of Ostwald-de Waele fluid over a variable thickness rotating disk with index decreasing are investigated by Xun [27]. Radiative flow due to stretchable rotating disk with variable thickness is addressed by Hayat et al. [28]. Srinivas et al. [29] studied MHD flow and heat transfer characteristics of Williamson nanofluid over a stretching sheet with variable thickness and variable thermal conductivity. Double diffusive flows over a stretching sheet of variable thickness with or without surface mass transfer is examined by Patil et al. [30].

Homogeneous-heterogeneous reactions occur in various chemically reacting systems having catalysis, combustion and biochemical systems. The powerful relationship between these two chemical reactions concerning with consumption and production of reactant species at different rates is quite complex on the catalyst surface and within fluid. Applications of such chemical reaction are in food processing, manufacturing of ceramics, hydrometallurgical industry, production of fog, dispersion and formation of polymer solutions and many others. Influence of homogeneous-heterogeneous reactions in the presence of stagnation point flow is examined by Chaudhary and Merkin [31]. Merkin [32] in another study revisited the study of isothermal heterogeneous-homogeneous reactions for flow of viscous liquid. Khan and Pop [33] scrutinized the heterogeneous-homogeneous reactions in flow of viscoelastic liquid over a stretched surface. Homogeneous-heterogeneous reactions in peristaltic flow with convective conditions is obtained by Hayat et al. [34]. A comparative study of casson fluid with homogeneous-heterogeneous reactions is presented by Khan et al. [35]. Hayat et al. [36] revisited the study of Cattaneo-Christov heat flux in Jeffrey fluid flow with Homogeneous-heterogeneous reactions.

Flow of silver-water nanofluid due to a rotating disk with variable thickness and chemical reactions is the main concern of present study. To our knowledge such attempts is not made so far. Thus the relevant problem is formulated. Convergent series solutions of governing equations are constructed by HAM [37–45]. Graphical results are used to elaborate the impacts of involved pertinent parameters on velocity, temperature, concentration, skin friction coefficient and Nusselt number. The main points are concluded.

2. Modeling

We consider axisymmetric nanofluid flow due to a rotating disk with angular velocity Ω. The disk of variable thickness i.e. at $z = d\left(\frac{r}{R_0}+1\right)^{-p}$ is taken. Temperature at the surface of disk is T_w and ambient temperature is assumed to be T_∞. Flow analysis is established with homogeneous–heterogeneous reactions of two chemical species $\widehat{A}$ and $\widehat{C}$. Beside this the effects of thermal radiation are not considered in the heat transfer analysis. For the cubic catalysis the homogeneous reaction is

$$\widehat{A} + 2\widehat{C} \rightarrow 3\widehat{C}, \quad \text{rate} = K_1\hat{a}\hat{c}^2, \tag{1}$$

and on the surface of catalyst the first order isothermal reaction is

$$\widehat{A} \rightarrow \widehat{C}, \quad \text{rate} = K_2\hat{a}, \tag{2}$$

where K_1 and K_2 are rate constants. Flow description is shown in Fig. 1.

Under the assumptions $\frac{\partial\hat{p}}{\partial r} = \frac{\partial\hat{p}}{\partial z} = 0$ and boundary layer approximation the constitutive equations are as follows [27]:

$$\frac{\partial\hat{u}}{\partial r} + \frac{\hat{u}}{r} + \frac{\partial\hat{w}}{\partial z} = 0, \tag{3}$$

$$\rho_{nf}\left(\hat{u}\frac{\partial\hat{u}}{\partial r} - \frac{\hat{v}^2}{r} + \hat{w}\frac{\partial\hat{u}}{\partial z}\right) = \mu_{nf}\left(\frac{\partial^2\hat{u}}{\partial z^2}\right), \tag{4}$$

$$\rho_{nf}\left(\hat{u}\frac{\partial\hat{v}}{\partial r} + \frac{\hat{u}\hat{v}}{r} + \hat{w}\frac{\partial\hat{v}}{\partial z}\right) = \mu_{nf}\left(\frac{\partial^2\hat{v}}{\partial z^2}\right), \tag{5}$$

$$(\rho C_\rho)_{nf}\left(\hat{u}\frac{\partial\widehat{T}}{\partial r} + \hat{w}\frac{\partial\widehat{T}}{\partial z}\right) = k_{nf}\left(\frac{\partial^2\widehat{T}}{\partial z^2}\right), \tag{6}$$

$$\hat{u}\frac{\partial\hat{a}}{\partial r} + \hat{w}\frac{\partial\hat{a}}{\partial z} = D_{\widehat{A}}\left(\frac{\partial^2\hat{a}}{\partial z^2}\right) - K_1\hat{a}\hat{c}^2, \tag{7}$$

$$\hat{u}\frac{\partial\hat{c}}{\partial r} + \hat{w}\frac{\partial\hat{c}}{\partial z} = D_{\widehat{C}}\left(\frac{\partial^2\hat{c}}{\partial z^2}\right) + K_1\hat{a}\hat{c}^2, \tag{8}$$

with boundary conditions

$$\begin{aligned}&\hat{u} = 0,\ \hat{v} = r\Omega,\ \hat{w} = 0,\ \widehat{T} = \widehat{T}_w,\\ &D_{\widehat{A}}\frac{\partial\hat{a}}{\partial z} = K_2\hat{a},\ D_{\widehat{C}}\frac{\partial\hat{c}}{\partial z} = -K_2\hat{a} \text{ at } z = d\left(\frac{r}{R_0}+1\right)^{-p},\\ &\hat{u}\rightarrow 0,\ \hat{v}\rightarrow 0,\ \hat{w}\rightarrow 0,\ \widehat{T}\rightarrow\widehat{T}_\infty,\ \hat{a}\rightarrow\hat{a}_0,\ \hat{c}\rightarrow 0 \text{ as } z\rightarrow\infty,\end{aligned} \tag{9}$$

where T is the temperature, $D_{\widehat{A}}$ and $D_{\widehat{C}}$ are diffusion coefficients, d is the thickness coefficient of the disk, R_0 the feature radius and p the disk thickness index a_0 is the positive dimensional constant, ρ_{nf} and

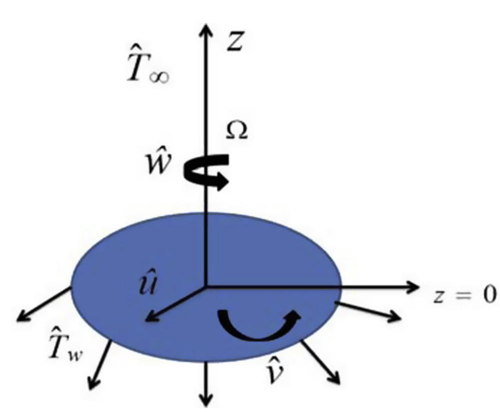

Fig. 1. Geometry of the problem.