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Numerical solutions for gyrotactic bioconvection of dusty nanofluid along a vertical isothermal surface

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ABSTRACT

The aim of the present paper is to establish the detailed numerical results for bioconvection boundary-layer flow of a two-phase dusty nanofluid. The dusty fluid contains gyrotactic microorganisms along an isothermally heated vertical wall. The physical mechanisms responsible for the slip velocity between the dusty fluid and nanoparticles, such as thermophoresis and Brownian motion, are included in this study. The influence of the dusty nanofluid on heat transfer and flow characteristics are investigated in this paper. The governing equations for two-phase model are non-dimensionalized and then solved numerically via two-point finite difference method together with the tri-diagonal solver. Results are presented graphically for wall skin friction coefficient, rate of heat transfer, velocity and temperature profiles and streamlines and isotherms. To ensure the accuracy, the computational results are compared with available data and are found in good agreement. The key observation from the present analysis is that the mass concentration parameter, D_ρ, extensively promotes the rate of heat transfer, Q_w, whereas, the wall skin friction coefficient, τ_w, is reduced by loading the dust parameters in water based dusty nanofluid.

1. Introduction

In fluid mechanics, bioconvection is a natural phenomenon which is primely related with the self-propelled microorganisms's suspension. It is worthy to mention here that, bioconvection is different from typical multi-phase flows, where particles behavior is not self-propelled; they are just carried by the fluid flow. Bioconvection originates due to instability in density stratification, which is created by directional swimming of microorganisms that are heavier than their surrounding fluid (*i.e.,* water). These self-propelled motile microorganisms tends to concentrate near the upper portion of the fluid layer and this accumulation makes the upper layer much denser than the lower region and ultimately produces instabilities into the system (for details see Refs. [1–6]). Bioconvection has numerous applications in biological and bio-microsystems. In addition, another potential application of theory of bioconvection is microbial-enhanced oil recovery, where nutrients and microorganisms are injected in layers of oil-bearing for correcting the permeability variation. Besides, the property of directional motion of motile microorganisms may be used for the concentration of cells, separation of dead and living cells, purification of cultures, or separation of various sub-populations [7–9]. However, bioconvection systems may be classified depending upon the directional movement of various species of microorganisms, but, they usually swim in upward direction (having larger density than the base fluid). For instance, (i) chemotaxis or oxytactic type of microorganisms swims upwardly due to the gradient of oxygen concentration, as they require certain amount of oxygen concentration to be active, (ii) gyrotactic microorganisms are the ones whose swimming direction is determined by making a balance in viscous and gravitational torques, and (iii) geotactic microorganisms swim against the gravitational effects [10,11]. In addition, the concept of nanofluid bioconvection has also a wide range of applications, for instance, nanomaterial processing, automotive coolants, sterilization process of medical suspensions and polymer coating. The idea of nanofluid bioconvection was first introduced by Kuznetsov [12,13], and later on numerous authors investigated the interaction of nanofluid with bioconvection (see Refs. [14–20]).

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The dynamics of gas-particle flows were of great interest in the past half century because of their wide spectrum of practical applications in atmospheric, engineering and physiological fields. For instance, fluidized beds, conveying of powdered materials, purification of crude oil, environmental pollutants, combustion chambers, petroleum industry are some of the applications of particulate suspensions [21]. The first experimental analysis on gas-particulate suspension flow was made by Farbar and Morley [22], and later on, Marble [23] developed the mathematical equations for dusty fluid flow problems. Singleton was the first to give the boundary-layer analysis for gas-particle flows, which is important to determine the accumulation of particles in suspension and their impingement on the surface [24]. Afterwards, several studies were done to give the physical insight to such two-phase dusty flows under different physical circumstances [25–30]. However, it is found that the problem of two-phase dusty nanofluid with gyrotactic bioconvection along a vertical surface has not been treated in the literature. Therefore, in the view of above discussion, present study is presented to investigate the influence of small solid particles on nanofluid bioconvection flow. It is assumed that (i) the gyrotactic microorganisms are self-propelled and (ii) the nanoparticles movement is due to the Brownian motion and thermophoresis and they are carried by the dusty fluid. On the basis of these physical assumptions, the interaction of microorganisms, dust particles and nanoparticles are expected to present an interesting problem in the area of fluid dynamics. The governing set of boundary-layer equations are converted into a convenient form through coordinate transformation known as primitive variable formulations (PVF). These nonlinear and coupled equations are solved numerically by using iterative finite difference method. The computational results are presented graphically in the form of streamlines and isotherms, velocity and temperature profiles, skin friction coefficient and rate of heat transfer by varying different physical parameters.

2. Flow analysis

Consideration has been given to the bioconvection boundary-layer flow of dusty nanofluid along a vertical isothermal wall. The surface of the wall is heated with temperature, T_w and it is assumed that $T_w \gg T_\infty$, where T_∞ is the ambient temperature of the base fluid. For detailed numerical simulations we have supposed that (i) the dust particles are of uniform size and equally distributed in water based nanofluid, (ii) nanoparticles does not affect the microorganism's swimming direction and velocity, (iii) bioconvection is only induced in purely sluggish cell suspension and (iv) Boussinesq approximation holds. In the light of the above conditions, the physical model describing the bioconvection flow can be written as (for details see Refs. [15,23,28]):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \Big(\rho g\beta(1-\phi_\infty)(T-T_\infty) - g(\rho_{np}-\rho)(\phi-\phi_\infty) - g\gamma(\rho_m-\rho)(n-n_\infty)\Big) + \frac{\rho_p}{\tau_m}(\bar{u}_p - \bar{u}) \tag{2}$$

$$\rho\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) + \frac{\rho_p}{\tau_m}(\bar{v}_p - \bar{v}) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) + \tau\left[D_B\left(\frac{\partial \phi}{\partial \bar{x}}\frac{\partial T}{\partial \bar{x}} + \frac{\partial \phi}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}}\right) + \frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial \bar{x}}\right)^2 + \left(\frac{\partial T}{\partial \bar{y}}\right)^2\right)\right] + \frac{\rho_p c_s}{\tau_T \rho c_p}(T_p - T) \tag{4}$$

$$\bar{u}\frac{\partial \phi}{\partial \bar{x}} + \bar{v}\frac{\partial \phi}{\partial \bar{y}} = D_B\left(\frac{\partial^2 \phi}{\partial \bar{x}^2} + \frac{\partial^2 \phi}{\partial \bar{y}^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) \tag{5}$$

$$\bar{u}\frac{\partial n}{\partial \bar{x}} + \bar{v}\frac{\partial n}{\partial \bar{y}} + \frac{\partial}{\partial \bar{x}}(n\bar{u}) + \frac{\partial}{\partial \bar{y}}(n\bar{v}) + \frac{bW_{mo}}{(\phi_w - \phi_\infty)}\left[\frac{\partial}{\partial \bar{x}}\left(n\frac{\partial \phi}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(n\frac{\partial \phi}{\partial \bar{y}}\right)\right] = D_{mo}\left(\frac{\partial^2 n}{\partial \bar{x}^2} + \frac{\partial^2 n}{\partial \bar{y}^2}\right) \tag{6}$$

$$\frac{\partial \bar{u}_p}{\partial \bar{x}} + \frac{\partial \bar{v}_p}{\partial \bar{y}} = 0 \tag{7}$$

$$\rho_p\left(\bar{u}_p\frac{\partial \bar{u}_p}{\partial \bar{x}} + \bar{v}_p\frac{\partial \bar{u}_p}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}_p}{\partial \bar{x}} - \frac{\rho_p}{\tau_m}(\bar{u}_p - \bar{u}) \tag{8}$$

$$\rho_p\left(\bar{u}_p\frac{\partial \bar{v}_p}{\partial \bar{x}} + \bar{v}_p\frac{\partial \bar{v}_p}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}_p}{\partial \bar{y}} - \frac{\rho_p}{\tau_m}(\bar{v}_p - \bar{v}) \tag{9}$$

$$\rho_p c_s\left(\bar{u}_p\frac{\partial T}{\partial \bar{x}} + \bar{v}_p\frac{\partial T}{\partial \bar{y}}\right) = -\frac{\rho_p c_s}{\tau_T}(T_p - T) \tag{10}$$

The corresponding boundary conditions are:

$$\bar{u}(\bar{x},0) = \bar{v}(\bar{x},0) = T(\bar{x},0) - T_w = \phi(\bar{x},0) - \phi_w = n(\bar{x},0) - n_w = 0,$$
$$\bar{u}(\bar{x},\infty) = \bar{v}(\bar{x},\infty) = T(\bar{x},\infty) - T_\infty = \phi(\bar{x},\infty) - \phi_\infty = n(\bar{x},\infty) - n_\infty = 0 \tag{11}$$

$$\bar{u}_p(\bar{x},0) = \bar{v}_p(\bar{x},0) = T_p(\bar{x},0) - T_w = 0,$$
$$\bar{u}_p(\bar{x},\infty) = \bar{v}_p(\bar{x},\infty) = T_p(\bar{x},\infty) - T_\infty = 0 \tag{12}$$

where $(\bar{u},\bar{v})$ are the components of the velocity field in $(\bar{x},\bar{y})$ direction, $\bar{p}$ the pressure, T the temperature, ϕ the nanoparticle concentration and n the microorganisms concentration for the fluid phase. Similarly, $(\bar{u}_p,\bar{v}_p)$ are $(\bar{x},\bar{y})$ components of velocity field, $\bar{p}_p$ the pressure and T_p the temperature for the particle phase. Further, ϕ_w is the nanoparticle volume fraction, n_w the density of microorganisms at the vertical surface, ϕ_∞ the ambient conditions at volume fraction of nanoparticles, n_∞ the density of the microorganisms, β the volumetric expansion coefficient of the base fluid (nanofluid), ρ the density of the base fluid, μ the dynamic viscosity, ρ_p the density of dust particles, ρ_{np} the density of the nanoparticles, ρ_m the microorganisms density, g the acceleration due to gravity, κ the thermal conductivity of the nanofluid, γ the average volume of a microorganisms, c_p the specific heat at constant pressure for base fluid, c_s the specific heat at constant pressure for the particles, τ_m the momentum relaxation time, τ_T the thermal relaxation time, D_B Brownian diffusion coefficient, D_T thermophoretic diffusion coefficient, b chemotaxis constant, D_{m0} diffusivity of microorganisms, W_{m0} the maximum cell swimming speed and $\tau = (\rho c)_p/(\rho c)_f$ the ratio of heat capacity of nanofluid to the heat capacity of the base fluid, respectively.