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The heat/mass transfer analogy for a backward facing step

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ABSTRACT

Heat and mass transfer from a surface to a stream of fluid are governed by Fourier's law and Fick's law respectively,which are mathematical manifestations of the process of diffusion. In the realm of transport processes, the mathematical equations describing the two phenomena can become analogous under certain assumptions and boundary conditions.

From an engineering perspective, it is difficult to measure heat transfer coefficients in separated flows because of high spatial thermal gradients and the intrusive nature of the various techniques. The analogous mass transfer measurement using the naphthalene sublimation, on the other hand, overcomes these challenges and presents significant advantages of speed, economy, better resolution and accuracy over its heat transfer counter-part. However, quantitatively, the diffusion rates of heat in air and naphthalene in air are different. So, the physical and mathematical similarity between the two processes can be utilized effectively only when the analogy factor ($F = Nu/Sh$) is determined.

This study investigates the heat/mass transfer analogy in a turbulent separated flow behind a backward facing step. The heat (Nu) and mass (Sh) transfer measurements were made using the thermal boundary layer measurement technique and the naphthalene sublimation measurement respectively under near identical flow conditions. Analogous boundary conditions of uniform temperature and constant concentration were imposed on the active surfaces, which is the recirculation-reattachment region behind the backward facing step.

The Nu and Sh values thus obtained were used to calculate the analogy factor, F, which was found to be 0.692.

1. Introduction

Turbulent separated flows are of fundamental importance because of their significant effects on heat, mass and momentum transfer. Ubiquitous in engineering applications viz. microelectronic circuit boards, combustors, heat exchangers, axial and centrifugal compressor blades and gas turbine blades, merits them a detailed study of their thermo-fluidic transport.

Detailed heat transfer measurements are often required in separated flows, such as in the design of turbine airfoils, where the knowledge of spatial variation in heat transfer coefficient can significantly aid in improving their performance. Extremely thin boundary layers near reattachment, which can be quite close to the point of maximum heat transfer coefficient, make the heat transfer measurements complex. In addition, the measurements may suffer from wall conduction and radiation errors.

An alternate way of obtaining the heat transfer information is to use the corresponding mass transfer analogies. Mass transfer measurements via naphthalene sublimation measurement, not only avoid these drawbacks, but provide higher spatial resolution with better speed and accuracy. The plausibility of such an indirect procedure stems from the fact that heat and mass transfer from a solid surface into a stream of fluid are both diffusion driven processes and can become analogous under certain sets of assumptions and boundary conditions, as discovered by Schmidt [1] and Nusselt [2] in the first half of the twentieth century.

$$\frac{D\theta}{Dt} = \frac{1}{RePr}\frac{\partial}{\partial \hat{x}_i}\left(\left(1 + \frac{\epsilon}{\nu}\frac{Pr}{Pr_t}\right)\frac{\partial \theta}{\partial \hat{x}_i}\right) \tag{1}$$

$$\frac{D\phi}{Dt} = \frac{1}{ReSc}\frac{\partial}{\partial \hat{x}_i}\left(\left(1 + \frac{\epsilon}{\nu}\frac{Sc}{Sc_t}\right)\frac{\partial \phi}{\partial \hat{x}_i}\right) \tag{2}$$

The non-dimensionalized heat transfer equation (Eq. (1)) and mass transfer equation (Eq. (2)) are similar but for the difference between Pr and Sc and Pr_t and Sc_t. Hence, under certain experimental conditions and equivalent boundary conditions, if $Pr = Sc$ and $Pr_t = Sc_t$ then the equations become identical and non dimensional

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Nomenclature

English symbols

D_{na}	diffusion coefficient of naphthalene vapor in air
F	heat/mass transfer analogy factor (Nu/Sh)
H	step height [20 mm]
h	convective heat transfer coefficient [W/m^2 K]
h_m	convective mass transfer coefficient [m/s]
k_1, k_2	arbitrary constants
k_{air}	thermal conductivity of air [W/m K]
M_n	molecular weight of naphthalene [kg/mol]
$\dot{m}$	naphthalene sublimation rate (convective contribution) [kg/s]
m, n	arbitrary constants
Nu	Nusselt number
Nu_{max}	maximum Nusselt number
$P_{v,w}$	naphthalene vapor pressure at wall [Pa]
Pr	Prandtl number
Pr_t	turbulent Prandtl number (ϵ/ϵ_H)
q_w	heat flux in normal direction at the wall [W/m^2]
$\overline{R}$	gas constant [J/mol K]
Re_H	Reynolds number based on step height, H
Sc	Schmidt number
Sc_t	turbulent Schmidt number (ϵ/ϵ_M)
Sh	Sherwood number
Sh_{max}	maximum Sherwood number
T	temperature [°C]
T_∞	freestream temperature [°C]
T_w	wall temperature [°C]
$T_{n,w}$	naphthalene wall temperature [°C]
t	time [s]
$\delta\tau$	time interval for forced convection mass transfer experiment [s]
$\Delta\tau_k$	time interval for natural convection of naphthalene [s]
u	streamwise velocity [m/s]
u^+	streamwise velocity in wall coordinates
u_∞	free stream velocity [m/s]
u_τ	friction velocity [m/s]
u'^+	velocity fluctuations in wall coordinates
x	streamwise coordinate [mm], $x = 0$ at the step
x_r	reattachment point [mm]
x_{max}	location of maximum Nu (Sh) [mm]
$\hat{x}_i$	non-dimensional coordinates
y^+	normal distance in wall coordinates
y	normal coordinate [mm], $y = 0$ along the plate
δy	naphthalene sublimation depth [m]
z	spanwise coordinate [mm], $z = 0$ along the center of the plate

Greek symbols

α	thermal diffusivity [m^2/s]
δ	boundary layer thickness [m]
δ_1	hydrodynamic Displacement Thickness [m]
δ_2	hydrodynamic Momentum Thickness [m]
ϵ	turbulent momentum diffusivity ($-\overline{u'v'}/\left(\frac{du}{dy}\right)$) [m^2/s]
ϕ	non-dimensional mass fraction in binary diffusion system = $\frac{\omega-\omega_\infty}{\omega_w-\omega_\infty}$
$\rho_{v,w}$	naphthalene vapor density at the wall [kg/m^3]
$\rho_{v,\infty}$	naphthalene vapor density in free stream [kg/m^3]
ρ_s	solid naphthalene density [kg/m^3]
θ	non-dimensional temperature = $\frac{T-T_\infty}{T_w-T_\infty}$
ω	mass fraction in binary diffusion system
ω_∞	freestream mass fraction
ω_w	mass fraction at the naphthalene surface

temperature, θ, and concentration, ϕ, have the same variation. This is the basic heat/mass transfer analogy which stems from the similarity of the two governing equations. Although $Pr_t = Sc_t$ is a good assumption for simple turbulent flow cases [3], it must be borne in mind that the equality of Pr and Sc is an exception rather than a commonality. The success of the aforementioned transformation with all its significant advantages [4] in case of different Pr and Sc, hinges on the existence and determination of a way to relate the two sets of transport coefficients (Nu and Sh) so that a closure to the solution is achieved.

There have been only a few studies to date to verify the existence of heat/mass transfer analogy. Sakamoto and Simon [5] used TEXSTAN simulations to calculate an analogy factor for GE90 and CF6 turbine blades for laminar flows. They found the analogy factor to be $F = 0.677$ for flow over a uniform temperature wall. Pachhapur [6] made heat transfer measurements on a turbine blade and compared them with earlier mass transfer measurements. He concluded that the analogy factor ranges between 0.49 and 0.64.

Han and Goldstein [7,8]were the first to explore the existence of heat/mass transfer analogy experimentally. They demonstrated the existence of a Colburn heat/mass transfer analogy for flow over turbine blades and end-walls. Kulkarni [9] experimentally verified the existence of Colburn analogy in laminar and turbulent boundary layers over a flat plate. For laminar and turbulent flows, for the range of Re explored in the study, the analogy factor was found to be 0.677 and 0.667 respectively. A uniform temperature boundary condition for the heat transfer experiments and an analogous constant concentration boundary condition for the mass transfer experiments were employed in his studies.

The present paper investigates the validity of the analogy in turbulent separated flow over a backward facing step (BFS). A BFS has been chosen for the study because of its geometrical simplicity and ability to capture the characteristic features of separated flows – separation, reattachment, recirculation and redevelopment of a boundary layer. While this geometry (Fig. 1) has been extensively studied in the past and often used as a benchmark case for computational fluid dynamics codes, there is no study in the literature, to the best of author's knowledge, that has investigated the Nu and Sh numbers under identical flow conditions and analogous boundary conditions. In this study, the heat and mass transfer coefficients have been obtained using the thermal boundary layer measurement technique and naphthalene sublimation measurement respectively. Analogous boundary conditions of uniform temperature and constant concentration in the recirculation, reattachment and redevelopment region were imposed.

2. Extended heat/mass transfer analogy, $Pr \neq Sc$

As discussed by Eckert et al. [10] an extended heat/mass transfer analogy can be developed if the following relations hold true:

1. The flow field is independent of Pr and Sc. In other words, while flow affects the thermal and mass transport, the converse is not true, i.e., the variation in Pr and Sc does not have an impact on the flow characteristics.