International Journal of Heat and Mass Transfer 113 (2017) 482–489

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Natural convection flow of a two-phase dusty non-Newtonian fluid along a vertical surface

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ARTICLE INFO

Article history:
Received 9 March 2017
Received in revised form 20 May 2017
Accepted 21 May 2017
Available online 3 June 2017

Keywords:
Natural convection
Dusty fluid
Two-phase
Non-Newtonian fluids
Modified power law
Vertical surface

ABSTRACT

The aim of this paper is to present a boundary-layer analysis of two-phase dusty non-Newtonian fluid flow along a vertical surface by using a modified power-law viscosity model. This investigation particularly reports the flow behavior of spherical particles suspended in the non-Newtonian fluid. The governing equations are transformed into non-conserved form and then solved straightforwardly by implicit finite difference method. The numerical results of rate of heat transfer, rate of shear stress, velocity and temperature profiles and streamlines and isotherms are presented for wide range of Prandtl number, i.e., $0.7 \leqslant Pr \leq 1000.0$, with the representative values of the power-law index n. A good agreement is found between the present and the previous results when compared for some special cases. The key observation from the present study is that the power-law fluids with ($n > 1$) are more likely to promote the rate of heat transfer near the leading edge.

1. Introduction

The interest in studying the dynamics of heat transfer problems involving non-Newtonian power-law fluids has been widely increased in the past half century, because of their wide range of usage in processing and manufacturing industries. For instance, most of the particulate slurries such as coal in water, synthetic lubricants, polymers, paints, emulsions, biological fluids such as blood, food stuffs such as jams, jellies and marmalades are few examples of fluids exhibit the non-Newtonian behavior. Although, several number of constitutive laws have been established to describe the behavior of non-Newtonian fluids, but the most deliberately used model in non-Newtonian fluid mechanics is the Ostwald-de Waele type power-law model (implied by [1]). Numerous researchers studied heat and mass transfer by taking into account power-law fluids. In this regard, Schowalter [2] applied the boundary-layer theory to shear thinning fluids (fluids for which power-law index is less than 1). After that Lee and Ames [3] extended the work of Schowalter [2] and established the similar solutions for power-law fluids. A theoretical analysis of laminar natural convection heat transfer on non-Newtonian fluids was conducted by Acrivos [4]. In that paper, the author investigated how the well-established expressions for the rate of heat transfer of Newtonian fluids can be generalized to include the non-Newtonian effects. A complete survey of the literature on non-Newtonian fluids is impractical however a few items are listed here to provide starting points for a broader literature (for details see Refs. [5–9]). In later years, Kawase and Ulbrecht [10] presented the approximate solution to the natural convection heat transfer from a vertical plate. Afterwards, Huang et al. [11] reported the influence of Prandtl number on free convection flow of power-law non-Newtonian fluids from a vertical plate. In [11], the authors presented similarity solutions and concluded that the average rate of heat transfer increases when Prandtl number rises. Later on, Kumari et al. [12] presented a theoretical analysis for laminar natural convection boundary layer flow of non-Newtonian power-law fluid. In that paper, the authors considered the vertical sinusoidal wavy geometry and established the numerical solutions via Keller-Box method for wide range of Prandtl number. Subsequently, a large amount of work for non-Newtonian fluids including integral, experimental, and numerical methods, was presented under various physical circumstances (see Refs. [13–17]).

In all the above-mentioned studies, attention has been given to fluids which are free from all impurities (clear fluid). But, pure fluid is rarely available in many practical situations, for instance, common fluids like air and water contains impurities like dust particles. Therefore, the analysis of the flow of fluids with suspended

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.05.080
0017-9310/

Nomenclature

C_f	skin friction coefficient
c_p	specific heat at constant pressure for fluid-phase
c_s	specific heat at constant pressure for particle-phase
D_ρ	mass concentration parameter
g	acceleration due to gravity
Gr	generalized Grashof number
K	dimensional empirical constant appeared in power-law
L	characteristic length
n	power-law index
Nu	Nusselt number coefficient
$\hat{p}$	dimensional pressure of carrier phase
$\hat{p}_p$	dimensional pressure of particle phase
p	dimensionless pressure of the carrier phase
p_p	dimensionless pressure of the particle phase
Pr	Prandtl number
Q	rate of heat transfer at the surface
T	dimensional temperature of fluid-phase
T_w	surface temperature
T_∞	ambient fluid temperature
T_p	dimensional temperature of particle-phase
ΔT	difference between surface temperature and ambient fluid temperature
$\hat{u}, \hat{v}$	dimensional fluid-phase velocity components
$\hat{u}_p, \hat{v}_p$	dimensional particle-phase velocity components
u, v	dimensionless fluid-phase velocity components
u_p, v_p	dimensionless particle-phase velocity components
$\bar{x}, \bar{y}$	dimensional cartesian coordinates
x, y	dimensionless coordinate system
U_c	reference velocity

Greek letters

α	thermal diffusivity
α_d	dusty fluid parameter
β	volumetric expansion coefficient
γ	ratio of c_p to c_s
κ	thermal conductivity
θ	dimensionless fluid-phase temperature
θ_p	dimensionless particle-phase temperature
ρ	density of fluid-phase
ρ_p	density of particle-phase
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
τ_m	velocity relaxation time of the particles
τ_T	thermal relaxation time of the particles
τ_w	shear stress at the surface

Subscripts

w	surface condition
∞	ambient condition

Superscripts

$-$	dimensional system
(i,j)	nodal positions

particles or gas-particle mixture have received notable attention due to its practical applications in atmospheric, engineering and physiological fields. Solid rocket exhaust nozzles, combustion chambers, blast waves moving over the Earth's surface, conveying of powdered materials, fluidized beds, environmental pollutants, petroleum industry, purification of crude oil and other technological fields are some of the practical applications of dusty fluids (see [18]). In this regard, Farbar and Morley [19] were the first to analyze the gas-particulate suspension on experimental grounds. After that, Marble [20] studied the problem of dynamics of a gas containing small solid particles and developed the equations for gas-particle flow systems. Singleton [21] was the first to study the boundary layer analysis for dusty fluid and later on several attempts were made to conclude the physical insight of such two-phase flows (see Refs. [22–30]) under different physical circumstances. In addition, Siddiqa et al. [32] reported the influence of thermal radiation on natural convection flow of contaminated air and water along the vertical wavy frustum of a cone. Very recently, the problem of compressible dusty gas along a vertical wavy surface was investigated numerically by Siddiqa et al. [33]. In that article, the authors solved the physical model numerically and reported the effect of compressibility, particulate suspension and sinusoidal waveform on rate of heat transfer and flow characteristics.

It is observed that most of the research related to two-phase (particle-fluid) flow assumes the fluid to be Newtonian in nature because such fluids have linear relationship between the shear stress and the shear rate. The applications of non-Newtonian power-law dusty fluids are found in process engineering, therefore it becomes important to reveal their flow characteristics. In this regard, Chamkha [34] studied the unsteady flow of a power-law dusty fluid with suction but the author did not considered the heat transfer phenomena. Thus, present work has been undertaken to give the more detailed analysis of the natural convection flow of a power-law dusty fluid by considering the thermal energy phenomena. The equations that govern the two-phase flow are reduced to a dimensionless form and then the coordinate transformation (primitive variable formulation) is employed to transform the two-phase boundary layer model into a convenient form. Since the equations are coupled and nonlinear, the solutions are obtained numerically by applying two point implicit finite difference method. The results for the two-phase problem are displayed in the form of wall shear stress, heat transfer rate, velocity and temperature profiles, streamlines and isotherms by varying several controlling parameters. The computational results for carrier phase are also compared with the published data of various studies and all agrees well with the present solutions. The effects of the presence of dust particle and the non-Newtonian nature of the fluids on flow and heat transfer characteristics are examined and discussed in detail. For full demonstration of the various non-Newtonian fluids, the behaviors of both Newtonian and dilatant fluids on the natural convection laminar flow along a vertical heated wall are studied by choosing the power-law index as $n = 1.0, 1.2, 1.5, 1.8, 2.0$.

2. Formulation of the problem

The physical model considered here is an isothermal vertical wall with a temperature, T_w, which is situated in two-phase dusty power-law fluid with ambient temperature, T_∞, such that $T_w > T_\infty$. In our detail computational work, the kinematic viscosity ν depends on shear-rate and is correlated by a modified power-law. Under the assumptions of two-phase flow given in [30,32], the governing equations for non-Newtonian, steady, laminar and incompressible fluid are given by (see Refs. [21,23]):

For the fluid phase:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \quad (1)$$