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Analysis of magnetohydrodynamic natural convection in closed cavities through integral transforms

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ABSTRACT

A hybrid numerical-analytical solution is proposed to analyze MHD (magnetohydrodynamic) natural convection of an electrically-conducting fluid within a square cavity, differentially heated at the sidewalls and subjected to an inclined external magnetic field. The first goal is to expand the spectrum of application of the so called Generalized Integral Transform Technique (GITT), dealing with a multiphysics formulation, while further demonstrating the relative merits of the proposed eigenfunction expansion approach in handling highly nonlinear and coupled systems of partial differential equations. The second goal is to provide a set of benchmark results in this important application for quantities of practical interest in determining the heat transfer rates, such as the average Nusselt number. The two-dimensional steady state equations are written in dimensionless form using the streamfunction-only formulation and are subsequently solved with the GITT approach, under automatic relative error control. Critical comparisons are performed against previous work reported in the literature, both computational and experimental, together with the corresponding physical interpretations, for different values of the governing parameters, such as Grashof number, Hartmann number, Prandtl number, and magnetic field inclination angle.

1. Introduction

Magnetohydrodynamics (MHD) deals with the motion of electrically-conducting fluids under the influence of externally applied electromagnetic fields. Examples of such fluids include ionized gases (plasma), liquid metals, saline water, and electrolytes. A quite comprehensive review on MHD may be found in the monograph by Davidson [1]. MHD is currently viewed as a particular case of a more general continuum mechanics-based theoretical framework referred to as Unified Electro-Magneto-Fluid Dynamics (EMFD) [2–5]. MHD natural convection inside closed cavities has received considerable attention in the past few decades because it occurs in numerous engineering applications such as in the liquid metal cooling of nuclear reactors and electric equipment [6–10], the manufacturing process of high-quality crystals [9,11–12], and magnetic-levitation casting [13], to name just a few.

The literature on MHD natural convection inside closed cavities is quite extensive and a detailed review is beyond the scope of the current work. The vast majority of previous studies focuses on two-dimensional laminar and incompressible flow of electrically-conducting fluids inside cavities, differentially heated either from the sidewalls or from its top and bottom walls, and subjected to either transverse, parallel or inclined magnetic fields with respect to the gravitational acceleration vector. Oreper and Szekely [12] were the first to numerically investigate the effect of an externally imposed magnetic field (transversal to gravity) on the natural convection inside a square cavity differentially heated from the sidewalls. Ozoe and Okada [11] investigated the MHD natural convection in three-dimensional cubic enclosures differentially heated from two vertical walls and under magnetic fields oriented along the principal axis of the cubic enclosure. Alchaar et al. [9] investigated the MHD natural convection inside a shallow cavity heated from below and cooled from the top, and subjected to an inclined magnetic field. Al-Najem et al. [10] investigated the MHD natural convection within a tilted square cavity differentially heated from its vertical walls and permeated by an inclined external magnetic field. Colaço et al. [14] revisited the MHD natural convection problem investigated in [10] and solved the MHD governing equations using a meshless method with radial basis functions (RBF) [15]. Results for the velocity and temperature

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Nomenclature

A_{ij}	integral coefficient given by Eq. (42a)
$\mathbf{B}_0^*$, $\mathbf{B}^*$, $\mathbf{B}'^*$	induced magnetic fields
B_{ij}	integral coefficient given by Eq. (42b)
C_{ijk}	integral coefficient given by Eq. (42c)
D_{ijk}	integral coefficient given by Eq. (42d)
$\mathbf{e}_{x*}$, $\mathbf{e}_{y*}$	unit vectors along the x^* and y^* axes
$\mathbf{E}^*$	electric field strength
E_{ijk}	integral coefficient given by Eq. (42e)
$\bar{f}_i$	transformed boundary condition given by Eq. (42i)
F_{ij}	integral coefficient given by Eq. (42f)
g	gravity acceleration
G_{ijk}	integral coefficient given by Eq. (42g)
Gr	Grashof number
$\mathbf{H}_0^*$	magnetic field strength
Ha	Hartmann number
H_{ijk}	integral coefficient given by Eq. (42h)
I_{ij}	integral coefficient given by Eq. (44c)
$\mathbf{J}^*$	electric current density
L	cavity length
Nu, $\overline{Nu}_{x=0}$	local and average Nusselt numbers, respectively
$N_{\Phi i}$, $N_{\Gamma i}$	normalization integrals for eigenfunctions $\Phi_i(y)$ and $\Gamma_i(y)$, respectively
N_ψ, N_T	truncation orders for the streamfunction and temperature fields, respectively
$p^*(x^*,y^*)$	pressure field
Pr	Prandtl number
Ra = GrPr	Rayleigh number
Re	Reynolds number
Re_m	magnetic Reynolds number
$T^*(x^*,y^*)$	temperature field
T_{ref}^*	reference temperature
T_C^*	temperature at the cold wall
T_H^*	temperature at the hot wall
$\bar{T}_i(x)$	transformed temperature
$\mathbf{v}^*(x^*,y^*)$	velocity vector field
v_0^*	reference velocity
x^*, y^*	Cartesian coordinates

Greek letters

α	fluid thermal diffusivity
α_i	eigenvalues for the streamfunction expansion
β	coefficient of thermal expansion of fluid
β_i	eigenvalues for the temperature expansion
γ	magnetic field inclination angle with respect to x-axis
σ_e	fluid electric conductivity
μ_e	fluid magnetic permeability
ν	fluid kinematic viscosity
ν_m	fluid magnetic diffusivity
ρ	fluid density
$\Phi_i(y)$	eigenfunction for the streamfunction expansion
$\Gamma_i(y)$	eigenfunction for the temperature expansion
$\bar{\Psi}_i(x)$	transformed streamfunction
$\psi^*(x^*,y^*)$	streamfunction field

Subscripts and superscripts

i,j,k	orders from eigenvalue problems
ref	quantity evaluated at reference temperature T_{ref}^*
$\sim$	normalized eigenfunctions
$*$	dimensional quantities
—	transformed quantities

distributions as well as for the average Nusselt number at the solid walls of the cavity have been reported in the literature, offering reference values for comparison and verification tasks. The effects of the governing parameters, namely, the Grashof number, the Hartmann number, the inclination angle of either the cavity or the magnetic field on the convective heat transfer rate, represented by the average Nusselt number, are well documented. The reported results mostly indicate that an external magnetic field, independent of its orientation, contributes to reducing the convective heat transfer through the cavity. The extent of heat transfer reduction depends strongly upon the imposed magnetic field strength. Magnetic fields oriented perpendicular to the heat flow direction are the most effective in suppressing convective heat transfer. For cavities heated from the bottom and cooled from the top, the results reported in [9] also indicate that the convection modes within the cavity depend strongly upon both the strength and inclination of the magnetic field.

From the literature review on MHD natural convection within closed cavities, the following remarks should be summarized. Firstly, the vast majority of the previous works relies on either finite-difference or finite-volume schemes to solve the governing equations. Such classical numerical schemes require spatial discretization of the domain and an approach to handle the velocity-pressure coupling. A few authors avoided the velocity-pressure coupling by rewriting the governing equations using the streamfunction-vorticity formulation. However, the boundary conditions adopted for the vorticity field at the solid walls are rarely reported, with a noteworthy exception in [11]. Secondly, few works have attempted to solve the governing equations using the streamfunction-only formulation [14], which has the advantage of not requiring boundary conditions for the vorticity field at the solid walls, albeit it requires a special scheme to accurately approximate fourth-order derivatives [14]. Thirdly, there are non-negligible discrepancies amongst the numerical results reported in the literature for the average Nusselt number, with relative deviations ranging from 2.9% to 32% [14]. Fourth, the majority of previous works reports numerical results only for the special case in which the magnetic field induced by fluid flow is negligible compared to the imposed one (inductionless approximation), decoupling Maxwell's equations from the Navier-Stokes equations for fluid flow. The current work comprises a detailed derivation of the conditions for this assumption to be valid.

Despite the extensive progress achieved by discrete numerical methods, analytical-type approaches for diffusion and convection-diffusion problems have been progressively advanced and extended, in part motivated by offering benchmark results for verification and calibration of the more flexible numerical methods. Powerful hybrid analytical-numerical schemes have emerged from the combination of classical analytical methods with modern computational methods for ordinary differential equations, benefiting as well from modern symbolic computation platforms. The Generalized Integral Transform Technique (GITT) is one such a hybrid method for solving linear or nonlinear diffusion and convection-diffusion problems, which has been developed for the last three decades, dealing with various classes of problems in heat and fluid flow, as reviewed in different sources [16–24]. A few contributions are here briefly mentioned, which have a closer connection to the problem under consideration. Natural convection inside cavities was first dealt with the GITT in [25], for a two-dimensional rectangular porous region with internal heat generation. Transient analysis of natural convection in porous cavities was then analyzed through the hybrid approach, both for