

Stochastic modeling of coupled electromechanical interaction for uncertainty quantification in electrostatically actuated MEMS

Nitin Agarwal, N.R. Aluru *

Department of Mechanical Science and Engineering, Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana-Champaign, 405 N. Mathews Avenue, Urbana, IL 61801, United States

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Abstract

This work proposes a stochastic framework based on generalized polynomial chaos (GPC), to handle uncertain coupled electromechanical interaction, arising from variations in material properties and geometrical parameters such as gap between the microstructures, applicable to the static analysis of electrostatic MEMS. The proposed framework comprises of two components – a stochastic mechanical analysis, which quantifies the uncertainty associated with the deformation of MEM structures due to the variations in material properties and/or applied traction, and a stochastic electrostatic analysis to quantify the uncertainty in the electrostatic pressure due to variations in geometrical parameters or uncertain deformation of the conductors. The stochastic analysis is based on a stochastic Lagrangian approach, where, in addition to uncertain input parameters and unknown field variables, the random deformed configuration is expanded in terms of GPC basis functions. The spectral modes for the unknown field variables are finally obtained using Galerkin projection in the space spanned by GPC basis functions. The stochastic mechanical and electrostatic analyses are performed in a self-consistent manner to obtain the random deformation of the MEM structures. Various numerical examples are presented to study the effect of uncertain parameters on performance of various MEMS devices. The results obtained using the proposed method are verified using rigorous Monte Carlo simulations. It has been shown that the proposed method accurately predicts the statistics and probability density functions of various relevant parameters.

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1. Introduction

The design and analysis of micro-electromechanical systems (MEMS) requires considering the interaction of various physical fields such as mechanical, electrical and possibly fluidic. In recent years, the advances in numerical simulation methods have increased the ability to accurately model these devices [1–4]. These simulation methods assume that the material properties and various geometrical parameters of the device are known in a deterministic sense. How-

ever, low cost manufacturing processes used for MEMS often result in significant uncertainties in these parameters which may lead to large variation in the device performance. Thus, in order to design reliable and efficient electrostatic MEMS devices, it is required to consider a stochastic analysis of the coupled electromechanical interaction.

The stochastic analysis of coupled electromechanical problem is complicated by two facts – firstly, due to the nature of the coupling between the mechanical and electrostatic energy domains, we need to deal with geometrical variations during the analysis, which is not straightforward, as it leads to random computational domains. Secondly, because of the multiphysics nature of the problem, uncertainties propagate from one energy domain to

* Corresponding author.

E-mail address: aluru@uiuc.edu (N.R. Aluru).

URL: <http://www.uiuc.edu/~aluru> (N.R. Aluru).

another, and need to be considered carefully. A framework for quantifying uncertainty for the electroosmotic and pressure-driven flow in a microchannel, involving coupled transport and electrostatic field equations, has been presented in [5]. In the past, the stochastic variations in various parameters during the design of electrostatic MEMS has been addressed using Monte Carlo (MC) simulations. The effect of various geometrical features on the design of a comb drive has been studied in [6] using Monte Carlo method incorporated in the ANSYS probabilistic design system (ANSYS/PDS). The variability in the performance of a ceramic MEMS actuator resulting from variations in the shape of the actuator and the air gap in the condenser, has been studied using MC simulations in [7]. A reliability-based analysis using the first-order reliability method (FORM) and design optimization framework for electrostatic MEMS is presented in [8]. Since the MC based methods are statistical in nature, their accuracy depends on the sample size. The simulations may become prohibitively expensive, especially for the analysis of MEMS devices, as it is expensive to solve the coupled electromechanical problem even in the deterministic case.

The most widely used non-statistical stochastic analysis technique is based on the approach pioneered by Ghanem and Spanos [9], known as *polynomial chaos*. The basic idea is to treat the uncertainty as a separate dimension (in addition to space and time) and to expand the field variables along the random dimension using polynomial chaos basis functions. Polynomial chaos is essentially a spectral expansion of the stochastic processes in terms of the orthogonal polynomials as given by Wiener's homogeneous chaos theory [10]. The homogeneous chaos expansion is based on *Hermite* polynomials and converges exponentially to any second-order random process, when the underlying random variables are Gaussian. This idea was further generalized by Xiu and Karniadakis [11], to obtain exponentially converging algorithms even for non-Gaussian random variables. This stochastic analysis technique based on polynomial chaos has been applied to model uncertainty in various problems such as computational mechanics [12,13], diffusion [14], fluid flow [15–17] and heat conduction [18,19].

This work presents a stochastic framework based on generalized polynomial chaos, to handle uncertain coupled electromechanical interaction, arising from variations in material properties and geometrical parameters such as gap between the microstructures, applicable to the static analysis of electrostatic MEMS. The proposed framework comprises of two components – a *stochastic mechanical analysis*, which quantifies the uncertainty associated with the deformation of MEM structures due to the variations in material properties and/or applied traction, and a *stochastic electrostatic analysis* to quantify the uncertainty in the electrostatic pressure due to variations in geometrical parameters or uncertain deformation of the conductors.

The stochastic analysis is based on a *stochastic Lagrangian approach*, where the random deformed configuration is

expanded as a sum of its mean, known as the mean or initial undeformed configuration, and a random deformation field applied to the conductors defined by the mean (or undeformed) configuration. For the stochastic mechanical analysis, in addition to the random deformation field, the uncertain input parameters such as Young's modulus, applied traction, etc. are expanded in terms of the generalized polynomial chaos (GPC) basis functions. These expansions are then used to compute stochastic analogues of various quantities that appear in the large deformation analysis. The uncertain deformation is finally obtained using Galerkin projection in the space spanned by GPC basis functions. The spatial discretization is done using standard finite element method (FEM). This methodology for quantifying uncertainty in large deformation problems is presented in [20], and has been applied to elasto-plasticity problems. For the stochastic electrostatic analysis, the random deformation field is used to obtain a *stochastic Lagrangian boundary integral equation* [21], which can be solved for uncertain surface charge density. The random surface charge density is discretized both in the random dimension and space using GPC and classical boundary element method (BEM), respectively. The computed uncertain surface charge density is then used to obtain the random electrostatic pressure. These stochastic mechanical and electrostatic analyses are performed in a self-consistent manner to obtain the random deformation of the MEM structures. We consider several numerical examples to study the effect of uncertain parameters on performance of various MEMS devices, highlighting the features of the proposed methodology. The results obtained using the proposed method are also verified using rigorous Monte Carlo simulations. It has been shown that the proposed method accurately predicts the statistics and probability density functions of various relevant parameters.

The paper is organized as follows: in Section 2 we present the most widely used spectral expansion method for random fields – the generalized polynomial chaos (GPC) expansion and various techniques required to employ GPC expansion as a stochastic discretization technique. In Section 3 we present the deterministic coupled electromechanical problem. In Section 4 we then present the stochastic electrostatic and mechanical analysis based on the stochastic Lagrangian approach and finally describe the formulation for the stochastic coupled electromechanical problem. In Section 5 we present some numerical examples to demonstrate the proposed methodology to quantify the effect of uncertain material properties and geometrical parameters on the performance of electrostatic MEMS devices. We finally conclude the discussion in Section 6.

2. Spectral stochastic representation

Let D be a domain in $\mathbb{R}^d$, $d = 1, 2$ and $\mathbf{x} \in D$. Let $(\Theta, B, \mathcal{P})$ denote a probability space, where Θ is the set of elementary events, B is the σ -algebra of events and $\mathcal{P}$ is the probability measure. The symbol θ specifies an

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