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A NURBS-based scaled boundary finite element method for the analysis of heat conduction problems with heat fluxes and temperatures on side-faces

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ABSTRACT

The scaled boundary finite element method (SBFEM) combined with isogeometric analysis (IGA) is proposed to solve the two-dimensional steady-state heat conduction problems in complex geometries. The main benefit of SBFEM is that the spatial dimension of analyzed domain is reduced by one and the solution is analytical in the radial direction. In this method, only the boundary of the computational domain requires discretization with finite elements leading to the reduction of computational efforts. However, SBFEM suffers from the finite element method related drawbacks. In the case of the complex geometric shapes, a large number of elements are necessary to obtain the exact representation of geometry in finite element method. Isogeometric analysis is a novel numerical technique based on the non-uniform rational B-splines (NURBS), where the geometry can be exactly represented. Moreover, this technique yields superior numerical accuracy, efficiency and convergence property in comparison to finite element method. In the proposed method, the segments of domain boundary with complex geometries are described with NURBS basis functions in IGA, while the straight segments of boundary are represented with polynomial basis functions as in the conventional SBFEM. Thus, the present approach combines the advantages of both SBFEM and IGA. The heat conduction problems of complex geometry can be more effectively handled with the proposed method considering the prescribed heat fluxes and temperatures on side-faces. The accuracy and efficiency of the proposed formulation are demonstrated by modeling five numerical examples involving the complicated geometry.

1. Introduction

The analysis of heat conduction problems extensively exists in many science and engineering fields including environmental science, chemistry, civil engineering, and cooling of electronic and mechanical equipment, etc. Due to its important significance in many engineering applications, the accurate and efficient calculation for this issue is essential in the study of the heat transfer. As the analytical solutions for the problems with complex geometry and material properties are generally not available, the numerical techniques require development and extension for the heat conduction problems. Thus, in recent years, various numerical methods have been developed to cope with the heat transfer problems ranging from the finite element method (FEM) [1–4], finite difference method (FDM) [5,6], boundary element method (BEM) [7,8], meshless method (MM) [9,10], and scaled boundary element method (SBFEM) [11,12]. There is also the element free Galerkin (EFG) method, which is applied to study the heat conduction problems by Singh et al. [13]. Wu et al. exploit the meshless local Petrov-Galerkin (MLPG) to analyze the steady-state heat transfer problems in two-dimensional space [14]. The reproducing kernel partial method (RKPM) is also applied to analyze the two-dimensional unsteady heat conduction problems [15].

Among all the numerical method mentioned above, the SBFEM, originally introduced by Wolf and Song [16], is a semi-analytical technique which inherits the main advantages of FEM and BEM

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with unique features of its own. This technique weakens the governing differential equations along the circumferential direction and solves analytically in the radial direction. In the conventional finite element analysis, the whole computational domain requires to be discretized. Nevertheless, only the boundary is discretized in SBFEM, which reduces the spatial dimension by one and spares the time in mesh generation and computing. Moreover, compared to BEM, no fundamental solution is required in SBFEM. Thus, SBFEM has been successfully applied to many practical problems, such as soil-structure interaction [17–22], fracture mechanics [23–25], piezoelectric materials [26–28], potential flow [29,30], sloshing problems [31–33], dynamics problems [34–37], elasto-plastic problems [38], acoustic problems [39], wave and porous interaction [40], electromagnetic problems [41], and seepage problems [42–44].

However, in the SBFEM, the boundary of computational domain is discretized with elements, which means the analysis model is generated by the approximation to the geometric model using the polynomials interpolation. Such an approximation between two models causes loss of accuracy and waste of time in mesh generation, especially for complex geometries, such as the smooth and curved shapes and circles. In order to reduce the error, a considerable number of elements are generally required to capture the exact representation in FEM.

Non-uniform rational B-spline (NURBS) is known as the standard technology for geometry representation in Computer-Aided Geometric Design (CAGD) because of its ability to exactly represent arbitrary free-form shapes in compact forms. Recently, the NURBS-based isogeometric analysis (IGA) developed by Hughes et al. [45] has turned out to be an efficient alternative to the classical FEM. The basic concept of IGA is to use the NURBS basis functions which can exactly represent the geometry for the numerical simulation. By utilizing the same NURBS basis functions for the geometry and field variables, the geometrical model described by NURBS can be direct applied to the analysis model without losing the exactness of geometry. No extra finite-element model that approximates the geometrical model is required. In addition, many desirable properties that NURBS basis functions process can be exploited to the numerical calculation. The higher degree of smoothness of NURBS can be applied to the high-order differential equations including plate and shell problems. Also any continuity of NURBS basis functions can be obtained by knot multiplicity, which is not the case for FEM. Due to these attractive merits, NURBS-based isogeometric analysis is applied with great success to the study of fluids [46], structures [47], turbulence [48], phase field modeling [49], fluid–structure interaction [50], contact problems [51], and optimization [52].

In order to inherit the advantages of SBFEM and IGA, the NURBS basis functions are employed to represent the complex boundary shape in the circumferential direction while the straight segments

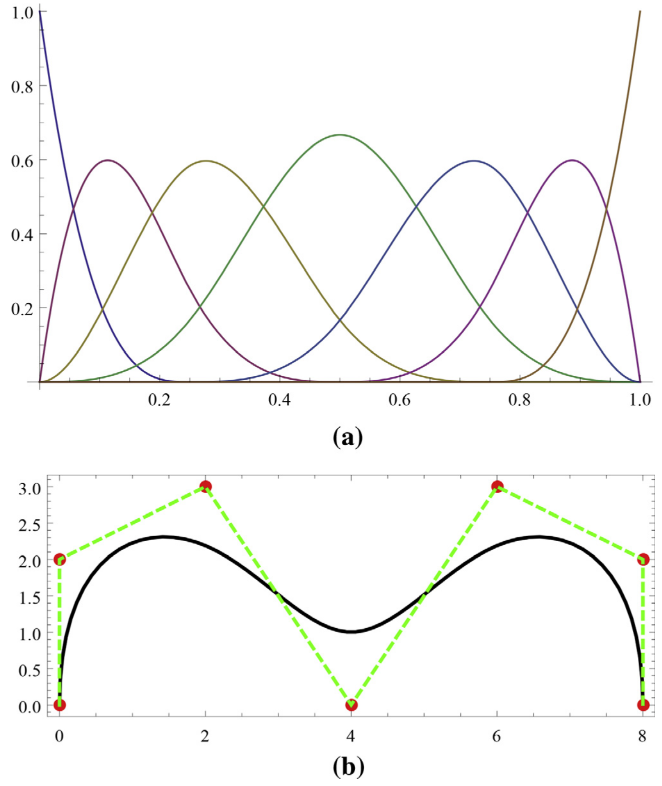

Fig. 1. (a) B-spline basis functions for $p = 3$ and open knot vector $\{0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1\}$. (b) B-spline curve with seven control points.

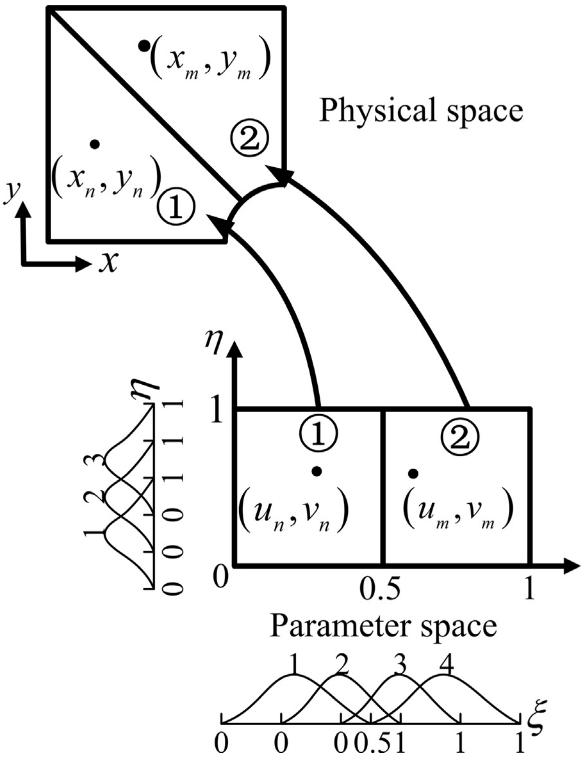

Fig. 2. NURBS mapping for a one-patch surface model.

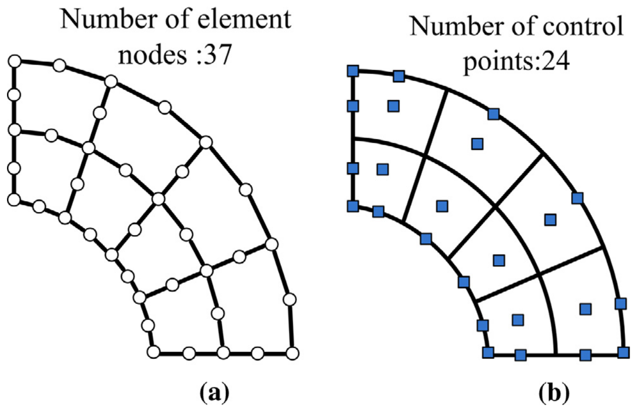

Fig. 3. A quadratic finite elements model of the conventional FEM and isogeometric analysis. (a) Finite elements of FEM. (b) Finite elements of isogeometric analysis.