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An analytical treatment for MHD mixed convection boundary layer flow of Oldroyd-B fluid utilizing non-Fourier heat flux model

Meraj Mustafa

School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan

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ABSTRACT

In current framework, an analytical treatment for mixed convection flow of an electrically conducting Oldroyd-B fluid adjacent to a vertical stretchable surface is provided. A non-Fourier heat flux approach is employed to formulate the energy balance relation. Using similarity approach, the governing equations are changed to a set of non-linear differential equations which are tackled by well-known analytical approach called homotopy analysis method (HAM). Appropriate range of auxiliary parameter is obtained by plotting the so called $\hbar$-curves. Velocity and temperature profiles are computed and elucidated in the existence of new physical mechanism, that is, thermal relaxation time in current research. The main implication of this research is that the relaxation and retardation times considerably alter the flow behavior near the surface. The results predict that heat penetration into the fluid reduces as the relaxation time of heat flux enlarges. Furthermore, the change in temperature gradient at the surface with increasing thermal relaxation time appears similar in magnitude at all considered Prandtl numbers. In assisting flow regime, we noticed a growth in velocity and temperature profiles for increasing strength of buoyancy force.

1. Introduction

Non-Newtonian flow behavior is prevalent in most of the chemical and allied processing industries. Viscoelastic fluids refer to those non-Newtonian liquids which display both viscous and elastic responses to deformation. In such liquids, shear stress is not only the function of instantaneous shear-rate but also the memory function of shear rate history. The shear-rate gradually diminishes with time once the shear stress is removed. Common examples include synthetic polymers, liquid crystals, coatings, inks, food products, ceramics, detergents, petroleum oil additives, molten plastics and biological fluids. Maxwell fluid model is considered as simplest viscoelastic model that combines viscous and elastic responses to deformation in terms of fluid relaxation time. Oldroyd-B fluid is another popular viscoelastic fluid model which has tendency to describe stress relaxation, creep and normal stress phenomena for many polymeric liquids. Bhatnagar et al. [1] provided series approximations for the flow of Oldroyd-B fluid near a deforming surface with variable free stream velocity. Oldroyd-B fluid flow in the region of stagnation-point on a stretchable surface was studied by Sajid et al. [2] using numerical approach. Shehzad et al. [3] reported analytical solutions for Oldroyd-B fluid flow caused by a bi-directional stretching sheet with temperature dependent thermal conductivity. Abbasbandy et al. [4] computed analytical solutions for Falkner-Skan flow of Oldroyd-B fluid along a stationary plate. HAM-Pade technique was opted to accelerate the convergence rate of series solutions. Motsa and Ansari [5] discussed numerical tackling for unsteady motion of Oldroyd-B fluid driven by an impulsively stretching plate. Also, Awad et al. [6] described the flow of Oldroyd-B nanofluid past a deforming sheet considering passive control of nanoparticle concentration at the boundary. They solved the governing equations of motion by spectral relaxation method. Effects of thermophoresis on the buoyancy assisting flow of Oldroyd-B fluid near a radiative surface were elucidated by Shehzad et al. [7] using analytical approach. Representative works in this direction can be sought through [8–13].

Heat transfer mechanism abounds in widespread industrial processes involving cooling towers, heat exchangers, space cooling, distribution of temperature/moisture over groove fields etc. Although the classical Fourier heat flux law [14] is preferred to model energy transfer in many practical situations but it has a major drawback that it does not fulfill the well-known causality principle. Cattaneo [15] came up with a generalization of the Fourier law by including the aspect of thermal relaxation time which is the time required to set up steady state heat conduction after the temperature difference is assigned. Mathematical framework of

E-mail address: meraj_mm@hotmail.com

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Cattaneo [15] gives rise to telegraph equation which has been found inadequate in describing heat transfer mechanism. Researchers have shown that the telegraph equation does not preserve the non-negativity of solutions and the maximum-minimum principle is not valid for the telegraph equation even in one-dimensional case [16–18]. Other physical inconsistencies can be found in the paper by Bright and Zhang [19] and in the book by Zhang [20]. To fulfill objectivity constraint, Christov [21] modified the Cattaneo equation by replacing the usual time derivative with the upper-convected derivative. Straughan [22] firstly used Cattaneo-Christov approach to analyze convective heat transfer in a Newtonian fluid flow. Uniqueness of solutions for some incompressible flow problems were proved by Tibullo and Zampoli [23] considering Cattaneo-Christov theory. As also stated in [22,23], Cattaneo-Christov model allows for the heat transport via propagation of thermal waves at finite speed. Such kind of heat transfer description via wave phenomenon rather than simply by diffusion has importance in applications including nanofluid flows and the modeling of skin burn injury. Haddad [24] figured out thermal instabilities associated with the Brinkman porous layer influenced by thermal relaxation time. Han et al. [25] analytically described the slip flow of viscoelastic fluid inspired by Cattaneo-Christov heat flux using homotopy approach. Their results indicate significant implications of thermal relaxation time on near-surface fluid temperature. Rotational effects in Maxwell fluid flow near a deforming sheet with Cattaneo-Christov heat conduction were described by Mustafa [26]. Characteristics of thermal relaxation time for Maxwell fluid flow bounded by an exponentially deforming non-isothermal surface were numerically explored by Khan et al. [27]. Hayat et al. [28] performed an analytical investigation for rotating flow of Jeffery fluid driven by deformable surface. Cattaneo-Christov model for Sakiadis flow of Maxwell fluid was examined by Abbasbandy et al. [29] utilizing two different numerical approaches. The influence of non-Fourier heat conduction for Sisko fluid flow caused by a permeable non-linearly deforming sheet was elucidated by Malik et al. [30] using homotopy approach. Some recent attempts in this direction can be stated through [31–35].

In order to make subsequent development in literature, we model the buoyancy effects on Oldroyd-B fluid flow which results from the stretching of a non-isothermal vertical surface. A novel Cattaneo-Christov heat flux approach is applied to formulate energy balance relation. Unlike previous studies, we establish correct formulation for magnetic and buoyancy force terms in case of Oldroyd-B fluid. Employing usual transformations, the local similarity equations are formed which are tackled by a reliable homotopy analytical approach suggested by Liao [36,37]. In HAM framework, the governing non-linear equations are transformed into infinite linear sub problems. HAM provides huge freedom to choose appropriate base functions and linear operators to approximate non-linear problem. This is important in the situation where convergence is largely dependent on the appropriate choice of initial guess. In contrast to the other analytical methods such as Adomian Decomposition Method (ADM), δ-expansion method and homotopy perturbation method (HPM), the convergence in HAM can be accelerated through an artificial convergence control parameter. It means that the method has a potential to tackle both weakly and strongly nonlinear problems. The impacts of emerging parameters are elucidated graphically in both assisting and opposing flow regimes. An important finding of this research is that thermal relaxation effects considerably alter the flow and temperature fields. A comparison of current computations with those of the published articles is also given in a limiting situation.

2. Mathematical modeling

Let us consider the flow of viscoelastic fluid obeying Oldroyd-B model adjacent to a heated or cooled vertical non-isothermal surface. We choose stationary Cartesian coordinate frame such that the coordinates x extends along the surface and y is normal to it. Flow is initiated due to stretching of the surface in vertical direction with velocity $u_w = ax$, where $a > 0$ denotes the stretching rate. It is assumed that wall temperature T_w varies with distance x according to $T_w(x) = T_\infty + bx$ in which b is constant whose value depends on the thermophysical properties of the fluid and T_∞ represents the quiescent fluid temperature (see Fig. 1). We take into account the Cattaneo-Christov model in order to incorporate the aspect of thermal relaxation time. The conducting Oldroyd-B fluid is permeated by uniform magnetic field strength B_0 (Tesla) normal to the stretching boundary. In low Reynolds number approximation, we ignore induced magnetic field in comparison with applied magnetic field. Electric field is assumed absent. In buoyancy induced flows, density variations cannot be totally neglected because they produce buoyancy gradients which are responsible for fluid motion. Here we utilize the Oberbeck-Boussinesq approximation [38,39] which involves two main assumptions. Firstly, the density ρ is a linear function of temperature T. Secondly, the density variations are assumed to be sufficiently weak such that they can be ignored everywhere in the governing equations with the exception that density differences are retained in the buoyancy term.

In view of the aforementioned assumptions, the boundary layer equations governing the mixed convection flow of Oldroyd-B fluid can be cast into the following forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = {} & \nu\frac{\partial^2 u}{\partial y^2} - \lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\partial y}\right) \\ & - \frac{\sigma B_0^2}{\rho}\left(u + \lambda_1 v\frac{\partial u}{\partial y}\right) \\ & + \nu\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) \\ & + g\beta_T\left[(T - T_\infty) + \lambda_1\left\{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{\partial u}{\partial x}(T - T_\infty)\right\}\right], \end{aligned} \tag{2}$$

$$\rho C_p(\mathbf{V}.\nabla T) = -\nabla\cdot\mathbf{q}, \tag{3}$$

where ν represents the kinematic viscosity of the fluid, λ_1 stands for fluid relaxation time, λ_2 for fluid retardation time, C_p represents the specific heat capacity and $\mathbf{q}$ the heat flux vector. The term in the square bracket in Eq. (2) represents the positive (upward) buoyancy force while the term with B_0 represents Lorentz force. Cattaneo-Christov model for heat flux is given by [21]:

$$\mathbf{q} + \lambda_3\left\{\frac{\partial\mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}\right\} = -k\nabla T, \tag{4}$$

in which k represents the fluid thermal conductivity and λ_3 denotes the thermal relaxation time. Heat flux vector $\mathbf{q}$ can be eliminated from Eqs. (3) and (4) to yield the following:

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \lambda_3\left\{\begin{array}{c} u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2} + 2uv\frac{\partial^2 T}{\partial x\partial y} + \\ \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y} \end{array}\right\} = k\frac{\partial^2 T}{\partial y^2}. \tag{5}$$