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Numerical model of thermal interactions between cylindrical cryoprobe and biological tissue using the dual-phase lag equation

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ABSTRACT

In the paper, the problem of biological tissue freezing is discussed. In contrast to the previously presented models based on the Pennes equation, the thermal interactions between the cryoprobe tip and soft tissue are described using the dual-phase lag model (DPLM). This model contains two delay times (the relaxation and thermalization times) and in this way, the finite velocity of the thermal wave is considered. The model of the freezing process is based on the introduction of a parameter called 'substitute thermal capacity' to the dual-phase lag equation. At the stage of numerical computations, the explicit scheme of the finite difference method is used. In the final part of the paper the examples of the computation are shown. A comparison with the solution resulting from the adoption of the Pennes equation and also the model verification with the experimental data are presented in the final part of the paper.

1. Introduction

Numerical modeling of biological tissue freezing can be a very effective tool supporting the preparation of cryosurgical treatment procedure. The results of computer simulation allow for the assumed input data to observe the time-dependent shape of frozen region and its dimensions, the temporary temperature distribution in the domain considered, etc.

The quality and reliability of the results obtained depends on the selection of the correct mathematical model of the process and the introduction of the adequate input data. So far, the model of thermal processes proceeding in the domain of soft tissue, as a rule, was based on the Pennes equation – e.g. [1–6]. As is well known, the Pennes equation (in the case of transient problem) is the parabolic PDE supplemented by two terms called the perfusion heat source and the metabolic heat source. The mathematical form of the perfusion heat source results from the assumption that the soft tissue is supplied by a big number of capillary blood vessels uniformly distributed in the tissue domain. The consideration of the large, thermally significant blood vessels requires the introduction of the so-called vascular models [7–9], but these problems will not be discussed here. The metabolic heat source can be treated as a constant value (but different for the different types of activity, e.g. rest, physical effort) or the temperature-dependent function [10].

It is well known that the Pennes' equation was based on the classical Fourier's law that depicted an infinitely fast propagation of a thermal wave. In reality, accumulating enough energy to transfer to the nearest element would take time in the process of heat transfer [11]. So, the lag time referred to as 'a relaxation time' was introduced by Cattaneo [12,13] and the appropriate energy equation (a hyperbolic PDE) is known as the Cattaneo-Vernotte equation. This equation is also used in the case of bioheat transfer. For example, the experimental investigation made by Roetzel et al. [14] showed that the value of relaxation time is of the order of 2 s for processed meat. In literature, the other values of this parameter can be also found (see: [15,16]).

Recently it is said that the better approximation of the bioheat transfer processes proceeding in the biological tissue domain can be obtained using the dual-phase lag approach. Generally speaking, the DPL model describes a macroscopic temperature wherein an inner microscopic tissue structure is taken into account by an introduction of two delay times to the energy equation (e.g. [17]).

The generalization of the Fourier law in which both the relaxation time τ_q and thermalization time τ_T appear leads to the dual-phase lag equation. The energy equation contains the second derivative of temperature with respect to time and also the mixed derivative both in time and space. The basic area of application of this equation is the micro-scale heat transfer. For example, if one considers the interactions between the ultrafast laser pulse and

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Nomenclature

c	volumetric specific heat [J/(m^3 K)]
C	substitute thermal capacity [J/(m^3 K)]
c_B	specific heat of blood [J/(kg K)]
h, k	geometrical mesh steps [m]
L	volumetric latent heat [J/m^3]
Q_m	metabolic heat source [W/m^3]
$\mathbf{q}$	heat flux vector [W/m^2]
R	radius of tissue domain [m]
R_1	cryoprobe radius [m]
S	frozen state fraction
T_1, T_2	border temperatures [°C]
T_B	arterial blood temperature [°C]
T_D	cryoprobe tip temperature [°C]
T_p	initial tissue temperature [°C]
w_B	blood perfusion rate [kg/(m^3 s)]
w_0	initial cooling rate [m/s]

Greek letters

λ	thermal conductivity [W/(m K)]
τ_q	relaxation time [s]
τ_T	thermalization time [s]

Subscripts

N	natural state
P	intermediate phase
F	frozen region

thin metal films then the Fourier parabolic equation is inadequate. Because of an extremely short duration, extreme temperature gradients and geometrical features of the domain considered, the lag times must be taken into account [17–20].

The number of papers concerning the DPL models applications in the scope of bio-heat transfer modeling is not so large. In 2005, Antaki published the paper [21] in which the DPL model has been used for the analysis of thermal processes proceeding in the processed meat domain. The problems of the heat conduction in the domain of living tissue subjected to an external heat source was analyzed by Liu [22]. In 2009, Liu and Chen [23] applied the DPL equation for the case of hyperthermia treatment modeling. The similar subject matter is discussed in the papers [24,25]. In particular, the authors consider the thermal damage to biological tissues caused by the laser irradiation. The various numerical aspects of DPLE solving are also analyzed. For example, in 2010, Majchrzak presented the solution of the dual-phase lag model of bioheat transfer using the general boundary element method [26], while the similar algorithm for 3D problems has been discussed in [27].

Recently, some works concerning the generalized dual-phase lag model based on the theory of porous media have appeared. The tissue is treated as a porous medium divided into two regions corresponding to the blood vessels and extravascular region (tissue). The relative proportions are determined by the parameter called 'a porosity'. The mathematical description is created by the DPL equation containing this parameter. The phase lag times are expressed in terms of the properties of blood and tissue, the interphase convective heat transfer coefficient and the blood perfusion rate [28]. Generally speaking, a two-temperature model is considered. The tissue temperature is determined by the GDPL equation, while the blood temperature results from the additional ordinary differential equation. In the specific case this model can be used assuming the constant blood temperature [28]. The two-temperature model has been successfully used for numerical modeling of the tissues heating (e.g. [29–33]).

The analysis of the strong tissue cooling leads first to freeze the extravascular medium [34–36]. So, this fact suggests to take into account different temperatures of freezing inside the biological cells and in the extravascular region. It seems, that the application of two-temperature models to describe the freezing process will allow for a more accurate analysis of heat transfer in human tissues subjected to the low temperatures [37].

In the case of freezing process modeling, the additional internal heat source controlling the evolution of freezing heat must be introduced [38,39]. As a rule, it is assumed that the capacity of this source is proportional to the local value of freezing rate [40,41].

The modeling of the freezing on the basis of the Pennes model can be realized in different ways. The comprehensive overview of the problems related to this process is presented in [42]. The mathematical model can be written in the classical form (the unknown function corresponds to the temperature) [43–45] but the enthalpy function (more precisely, the mixed enthalpy-temperature approach [46,47] and the Kirchhoff transformation [48] can also be considered. One can find the works (e.g. [46]) in which the model of freezing proceeding in the interval of temperature is substituted by the model concerning the pure substances (the Stefan problem). The freezing front corresponds, as a rule, to the border temperature between the frozen region and the intermediate zone. It seems that this temperature should be defined in any other way using the generalized theorem about the mean value of definite integral [38].

In the opinion of the authors of this paper, the most effective and simple at the stage of numerical modeling is the approach called the 'one domain method', also known as a 'fixed domain method'. The essence of this method is a certain way of joining the internal heat source resulting from the freezing process to the left hand side of the energy equation and then in the place of volumetric specific heat the parameter $C(T)$ called 'a substitute thermal capacity' (STC) appears. The STC is especially convenient in the case when the phase change proceeds in an interval of temperature (biological tissues, alloys, solutions, etc.) The problems of the above parameter definition are discussed in detail in [38,39,49,50]. Generally speaking, one can assume the temperature-dependent function describing the local and temporary frozen state fraction S (T) between the border isotherms at the neighborhood of the point considered. The knowledge of this function allows one to define the STC (see: next chapter). The other approach depends on the 'a priori' assumption of the mathematical form of STC (e.g. a bell-type function), but this function must fulfill the condition concerning the equality of the STC integral between the border temperatures and the change of physical enthalpy resulting from the evolution of the volumetric freezing heat and the cooling of the intermediate phase from the upper to lower temperatures liming the intermediate phase sub-domain [38]. The substitute thermal capacity for the intermediate zone subdomain is, as a rule, assumed in the form of the bell-type function (e.g. [39,51]) or as the constant value being the sum of the intermediate zone volumetric specific heat and the so-called spectral latent heat (e.g. [38]).