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Nondestructive evaluation of spatially varying internal heat transfer coefficients in a tube

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ABSTRACT

We derive a rule for the reconstruction of the internal heat transfer coefficient h_{int} of a pipe, from temperature maps collected on the external face. The pipe is subjected to internal heating by connecting two electrodes to the external surface. To estimate h_{int} we apply the perturbation theory to a thin plate approximation of a boundary value problem for the stationary heat equation.

1. Introduction

The present paper deals with the *inverse heat conduction problem* (IHCP) in the form studied in [4,6].

Let $S = \{(r\cos(\phi), r\sin(\phi)); r \in [L-a, L]; \phi \in [0, 2\pi)\}$ be the cross section of a helically coiled cylindrical tube C of radii L (external) and $L-a$ (internal) with $a \ll 2\pi L$. Actually, C is both a thermal and electrical conductor like the stainless steel type AISI 304 one considered in [4]. The experimental set up is fully described in [4] and summarized in the next paragraph.

Several engineering applications involve curved pipes carrying a moving fluid, as it happens in heat exchangers or in devices used for transferring heat in heat engines and in industrial equipment. Therefore, the understanding of the relationship among the dynamic properties of a fluid flowing in a pipe (in particular its velocity profile) and the resulting temperature distribution inside it is of great importance. The first theoretical study concerning the flow in a curved pipe is dated back 90 years [9], but a systematic study of heat transfer in curved pipes has been conducted 40 years later, both for a fluid in the laminar regime [16] and in the turbulent region [17]. The problem of heat transfer in curved tubes has been recently the subject of a review paper [19]. In essence, the main effect of curvature is the emergence of a secondary flow due to centrifugal forces. This in turn produces a deformation of the velocity field (parabolic in the laminar regime in a straight tube) shifting the maximum axial velocity towards the outer side of the pipe bend. As a consequence, the temperature distribution is distorted alike.

The conductor C is subjected to a uniform heat generation Q obtained by the Joule effect connecting a couple of electrodes located at its ends. A fluid fills the internal helicoidal cylinder of radius $r = L - a$ and flows smoothly in it. The fluid exchanges heat with the conductor C through its inner surface. To minimize the heat exchange with the environment, C was thermally insulated. A small portion of the external tube wall was made accessible to an infrared camera by removing the insulating layer, and it was coated by a thin film of opaque paint of uniform and known emissivity.

The problem, formulated in [4,6], consists in recovering the internal *heat transfer coefficient* (HTC) from thermal maps collected on the accessible portion of the external face of C. In the case of a laminar flow along an helically coiled cylindrical cavity, we observe that the HTC is essentially the same for all orthogonal sections of C so that, as pointed out in [4], the model is intrinsically two dimensional and our analysis can be reduced to the section S only. Furthermore, the HTC deviates from a background value (corresponding to the case of the straight cylinder) and takes an asymmetrical shape determined by the fluid dynamics inside C.

The temperature of the specimen reaches a reasonably stationary regime for $t \geqslant T_{lim}$ whose value is related to the diffusivity of the tube C. For this reason, as suggested in [4], we focus our atten-

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Nomenclature

Parameters

C	helically coiled cylindrical tube
S	cross section of C
L	external radius of S
$L-a$	internal radius of S
$\epsilon = \frac{a}{L}$	thickness-to-radius ratio
ah^{int}	internal heat transfer coefficient
U_{int}	internal temperature
ah^{ext}	external heat transfer coefficient
U_{ext}	external temperature
T_{lim}	time required to reach stationary regime
Q	constant heat generated by a couple of electrodes
$\tilde{\Omega} = [0, 2\pi) \times [0, 1]$	image of S after normalization of z
κ	thermal conductivity
$\tilde{\kappa} = \frac{\kappa}{L^2}$	normalized conductivity
h_k	coefficients of the expansion of h^{int} in powers of ϵ
u_k	coefficients of the expansion of u in powers of ϵ

Acronyms

BVP	Boundary Value Problem
IHCP	Inverse Heat Conduction Problem
HTC	Heat Transfer Coefficient
IP	Inverse Problem
TPA	Thin Plate Approximation
FEM	Finite Elements Method
LOESS	LOcal regrESSion

tion to a stationary *boundary value problem* (BVP) for Poisson's equation in S which models the steady-state energy balance.

In cylindrical coordinates, the BVP is made up by Laplace's equation in polar coordinates

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\phi\phi} + \frac{Q}{\kappa} = 0 \tag{1}$$

in S and the boundary conditions

$$\kappa u_r(\phi, L) + ah^{ext}(u(\phi, L) - U_{ext}) = 0 \tag{2}$$

$$-\kappa u_r(\phi, L-a) + ah^{int}(u(\phi, L-a) - U_{int}) = 0 \tag{3}$$

for $\phi \in [0, 2\pi)$.

Here, κ is the thermal conductivity of C, ah^{int} and ah^{ext} are the heat transfer coefficients (internal and external respectively). The temperature of the outer environment is U_{ext} while the bulk (internal) temperature is U_{int}. We are able to take temperature maps of the external surface by means of an infrared cameras. In our 2D context, these maps reduce to positive functions of one angular real variable $\phi \in [0, 2\pi)$.

We refer to the BVP (1)–(3) as to the *direct model* under which our inverse problem will be formulated. More precisely, in this paper we produce an approximated explicit solution of the following inverse problem:

IP. Suppose that we know the geometrical parameters a and L, the heat source Q, the external heat transfer coefficient ah^{ext}, the conductivity κ. Given one temperature map $\bar{u}(\phi)$ of the external boundary of S, we must identify the internal heat transfer coefficient ah^{int}.

The BVP (1)–(3) has a unique stable solution when $\kappa, Q, U_{int}, U_{ext}, ah^{int}$ and ah^{ext} are known (see for example [21]). In Appendix we prove that the inverse problems **IP** has a unique solution.

Problem **IP** belongs to the quoted class of IHCP which, in turn, is included in the wider family of inverse problems in partial differential equations. This kind of problems are approached usually through regularized optimization (compare also [4,6]). Here we choose a different road and use the thinness of S to expand temperature u and unknown h^{int} in powers of $\epsilon = \frac{a}{L}$. In this way, we obtain a perturbative hierarchy from which we derive a formal explicit approximation of HTC.

Methodological and bibliographical remark. A very complete and up-to-date book about regularization of linear and nonlinear inverse problems is [10]. In our opinion, useful references about theoretical foundations and different way to solve IHCP are: [20] (mathematical foundation of ill-posedness of IHCP), [11] (numerical analysis of Cauchy problem for heat equation), [3] (the fundamental book about IHCP), [2] (seminal paper in the field of thermal imaging of unknown boundary), [18] (still very useful book about numerical methods for IHCP, each chapter include complete references at the date), [15] (thin plate approximation to detect damaged boundaries), [1] (a rich survey about optimization and iterative methods for IHCP), [14] (thin plate approximation to recover HTC in a simplified framework), [7] (a stability estimate for HTC identification), [5] (application of domain derivative to boundary identification), [22] (stability estimates), [12] (variational methods to estimate HTC).

1.1. Short description of our procedure and results

Since we assumed $a \ll 2\pi L$, any solution of (1)–(3) can be regarded as a function of a small adimensional parameter $\epsilon = \frac{a}{L}$. Since we are interested in the MacLaurin series of u in ϵ, it is helpful to change variables in order to define u in a rectangular domain that does not depend on a. Hence, we shift and rescale r so that the radial variable is transformed into the cartesian vertical one $\zeta = \frac{L-r}{a}$. Also, we have $\frac{\partial u}{\partial r} = -\frac{1}{a}\frac{\partial u}{\partial \zeta}$ and $\frac{1}{r} = \frac{1}{L(1-\epsilon\zeta)}$. The domain S in the new variables is the rectangle

$$\tilde{\Omega} = [0, 2\pi) \times [0, 1]$$

where (4)–(6) becomes

$$u_{\zeta,\zeta} - \epsilon\frac{1}{(1-\epsilon\zeta)}u_\zeta + \epsilon^2\frac{u_{\phi\phi}}{(1-\epsilon\zeta)^2} + \epsilon^2\frac{Q}{\tilde{\kappa}} = 0 \tag{4}$$

$$\tilde{\kappa}u_\zeta(\phi, 1) + \epsilon^2 h^{int}(u(\phi, 1) - U_{int}) = 0 \tag{5}$$

$$-\tilde{\kappa}u_\zeta(\phi, 0) + \epsilon^2 h^{ext}(u(\phi, 0) - U_{ext}) = 0 \tag{6}$$

with adiabatic periodic conditions $u_\phi(-\pi, \zeta) = u_\phi(\pi, \zeta) = 0$ for $\zeta \in [0, 1]$ on the vertical sides of $\tilde{\Omega}$. Here and in what follows, $\tilde{\kappa} = \frac{\kappa}{L^2}$.

We expand u and h^{int} in powers of ϵ and plug them into the BVP (4)–(6). In this way we obtain a perturbative hierarchy of relations amongst their coefficients. We solve these relations with respect to the coefficients of h^{int}. This procedure is called thin plate approximation and is borrowed from [14] where it was applied to a similar problem for Laplace's equation in a thin rectangle.

In Section 2, we derive explicitly $h_0^{int} \dots h_3^{int}$ so that the following explicit finite expansion of the internal heat transfer coefficient

$$ah^{int}(\phi) = ah_0^{int}(\phi) + a\epsilon h_1^{int}(\phi) + a\epsilon^2 h_2^{int}(\phi) + a\epsilon^3 h_3^{int}(\phi) + O(\epsilon^4) \tag{7}$$