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Non-linear interaction of buoyancy with von Kármán's swirling flow in mixed convection above a heated rotating disc

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ABSTRACT

In this paper, a systematic computational and theoretical study of the thermo-fluid-dynamics governing the flow above a heated horizontal rotating disc is presented. The fluid flow field is much more complex here as compared to von Kármán's original solution (which took into account only the effect of disc rotation), and the effects of non-linear interaction between buoyancy and rotation are critically analysed here by studying the separate and combined roles of disc rotation and buoyancy on the fluid dynamic and heat transfer characteristics. The self-similarity of von Kármán's flow field is lost, and the present paper establishes, for the first time, that the flow field above a heated rotating disc is divided into three distinct fluid dynamic regions. The three regions are demarcated by the loci of $\hat{V}_z = 0$ and $\hat{V}_r = 0$. In region 1 (R-1), $\hat{V}_r$ is positive and $\hat{V}_z$ is negative (such directions of the velocity components are characteristic of von Kármán's flow or pure forced convection). In region 2 (R-2), $\hat{V}_r$ is negative and $\hat{V}_z$ is positive (such directions of the velocity components are characteristic of pure natural convection near a static disc surface). In region 3 (R-3), both $\hat{V}_r$ and $\hat{V}_z$ are negative. The forced convection results are obtained asymptotically at a large non-dimensional radius $\mathfrak{R}$ within the region R-1 showing the dominance of forced convection mechanism, however, the fluid retains the signature of natural convection even at large values of $\mathfrak{R}$ in the region R-3 where there is an inward radial velocity. Similarly, although a plume forms in the central portion of the disc where the solution is dominated by the effects of buoyancy, the fluid retains a signature of the disc rotation in the helical pathlines of fluid particles rising in the plume (whereas there is no swirl velocity present in pure natural convection above a static disc). The non-linear interaction between buoyancy and rotation results in several peculiar, rather non-intuitive, flow phenomena. Examples of such peculiarity include (i) the presence of a very sensitive spot on the upper boundary such that for a small change in this initial position the fluid pathlines may face drastically different final outcomes, (ii) the presence of a small portion near the centre of the disc where the fluid supplies energy to the disc, (iii) the effect of rotation on the rate of convective heat transfer being diminished by buoyancy over certain part of the disc while being enhanced over another part. This non-linear effect on Nusselt number is quantified here in terms of a Grashof number defined for mixed convection (Gr_{mc}).

1. Introduction

The study of fluid flow near a rotating disc is important for both its scientific value and engineering application. The famous fluid dynamicist von Kármán provided the first analysis [1] in this field by developing a similarity solution for the flow due to a semi-infinite rotating disc. The similarity solution was established for steady, laminar, incompressible and axi-symmetric flow. Guha and Sengupta [2] has given a lucid physical description of how all three components of the velocity vector, viz. tangential (V_θ), radial (V_r) and axial (V_z) components, arise in von Kármán's swirling flow. The tangential component is a direct consequence of the disc rotation, whereas the (outward) radial component is an indirect effect. A steady axial flow from the ambient towards the disc occurs to supply the steady radial efflux. An important distinctive feature of a rotational boundary layer is that it, instead of growing continuously like the case of the flat-plate boundary layer, plateaus to a finite thickness which is proportional to $\sqrt{\nu/\Omega}$ where Ω is the angular velocity and ν is the kinematic viscosity of the fluid.

Since the publication of von Kármán's paper, intensive research interest in this field has continued for nearly a century and still new papers are being published. Zandbergen and Dijkstra [3] provided a detailed review on von Kármán's swirling flow. Some

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recent advances can be found in references [2,4–10]. A description of the contribution of great fluid dynamicists like Ekman [11], Batchelor [12], and Stewartson [13], and, a review of related studies on rotational boundary layer are given in reference [14].

In this paper, the thermo-fluid-dynamics of the flow above a heated horizontal rotating disc is studied. The fluid flow field becomes more complex than von Kármán's swirling flow described above because of the mutual interaction between fluid flow and heat transfer that results into mixed convection. The physical configuration is shown in Fig. 1. r (radial), θ (azimuthal) and z (axial) are the three coordinates and O is the origin of the cylindrical coordinate system. The disc is rotating about the z-axis at an angular speed Ω. The disc-surface is located at $z = 0$ and the solutions given here correspond to $z \geqslant 0$. The upper surface of the disc is at uniform temperature, which is greater than that of the ambient fluid. The lower surface of the disc is insulated.

Previous papers related to the effect of heat transfer on von Kármán's swirling flow are either theoretical or experimental. A good survey of such work can be found in the review article by Dorfman [15], and also in the monographs written by Kreith [16], Owen and Rogers [17], and Shevchuk [18]. Cobb and Saunders [19] reported an experimental study of heat transfer near a vertically aligned rotating disc, while Elkins and Eaton [20] performed experiments on heat transfer above a heated horizontal rotating disc. Since the experimental set-up and the thermal boundary conditions used in ref [20] correspond to the theoretical study undertaken here, it would be pertinent to give an outline of the experimental procedure here. Elkins and Eaton [20] used a 1 m diameter and 1 mm thick disc. The material of the disc was 304 stainless steel. Below the disc, Kapton thermofoil heaters were attached; and, below the heater, an insulating balsa wood disk 15 mm thick was attached. For structural support a 15 mm thick aluminium disc was attached to the balsa. The measurement of the disc surface temperature could be made throughout the disk using 40 copper-constantan thermocouples. To maintain uniform disc-surface temperature, the disc was split into three different annular regions. The inner disk with a 7.5 cm radius was unheated. The region from 7.5 cm to 20 cm radius was heated by the inner heater and from 20 cm to 48 cm by the outer heater. A control system was used to ensure the uniformity of temperature. It is shown later in the paper (Section 2.6) that the present theoretical prediction of the temperature profile in the forced convection region matches well with the experimental values given by Elkins and Eaton [20]. However, no detailed temperature measurement in the mixed convection region is available in [20].

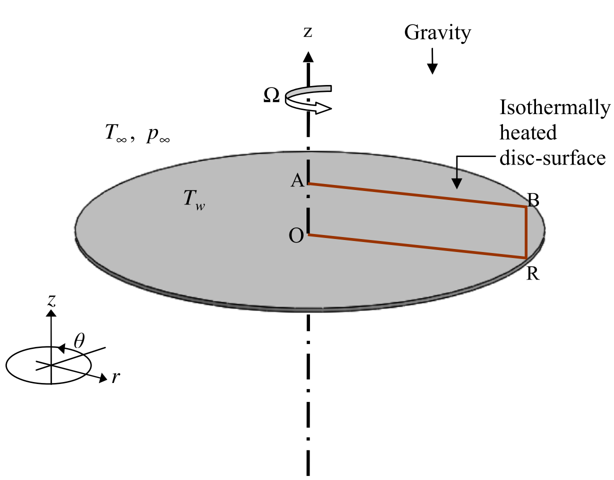

Fig. 1. (Colour online) Schematic diagram of the physical configuration (OABR is a two dimensional computational domain which is used to determine a three-dimensional, axi-symmetric flow field).

Launder and Sharma [21] measured the critical Reynolds number required for the transition from laminar to turbulent flow (details given later). Experimental observations and related discussion on the onset of transition for flow above a rotating disc without heat transfer can be found in Lingwood [22]. For forced convection above a heated rotating disc (i.e. neglecting the effect of natural convection), Wagner [23] derived a theoretical expression for the coefficient of heat transfer considering laminar flow of air. In his paper, Wagner [23] considered approximate profiles for V_r and V_θ within the boundary layer. Theoretical studies on pure natural convection ($\Omega = 0$) above an isothermally heated disc can be found in references [24–26], and interesting discussion on natural convection above a flat, rectangular surface can be found in references [27–30].

A theoretical analysis of mixed convection above a heated rotating disc is presented in a recent paper [31], in which the temperature of the disc-surface (T_w) was assumed to vary quadratically with the radial distance from the disc-centre so that a theoretical solution was possible. Such a temperature boundary condition is restrictive from a practical point of view. We could not find any previous work on mixed convection above a heated rotating disc that is directly relevant for the present study.

1.1. Objective of the present work

The objective of the present paper is to provide a systematic computational fluid dynamic (CFD) study of the laminar mixed convection above an isothermally heated, horizontal rotating disc. As far as we know, a detailed computational study on this topic is not available in the previous literature.

The second objective of this work is to explore and critically expound the detailed fluid dynamic features of mixed convection above a heated rotating disc, and, to express the results in terms of appropriate non-dimensional variables. The fluid dynamics depends on non-linear interaction between buoyancy and rotation, and all the subtleties cannot be quantitatively captured by theoretical (analytical) methods. This establishes the utility of the computational approach.

Finally, the physical understanding is enhanced by studying the separate and combined roles of disc rotation and buoyancy on the fluid dynamic and heat transfer characteristics. A limiting case, in which the effect buoyancy is absent, is devised here by setting $g = 0$ in the CFD simulations. This limiting case represents forced convection for which a similarity theory is also presented. While comparing the results of mixed convection with the results of forced convection, the special effect of buoyancy can be appreciated.

1.2. An early glimpse of the fluid dynamics revealed

The details of the fluid dynamics can be understood only after all the results are presented and explained. However, a brief qualitative overview of the new physics is introduced here in the hope that the conceptual summary given in Fig. 2 will be helpful to interpret the quantitative details presented later. In the original von Kármán's swirling flow, the tendency of a fluid particle from the ambient is to be drawn towards the rotating disc ($V_z \leqslant 0$) while moving radially outward ($V_r \geqslant 0$). On the other hand, if the disc is static ($\Omega = 0$) but the disc-surface temperature (T_w) is greater than the temperature of the quiescent fluid (T_∞), then a purely natural convective flow is established. In such flow field, the tangential component of velocity does not exist ($V_\theta = 0$), the radial velocity is inward ($V_r \leqslant 0$), and the axial velocity (near the disc surface) is away from the disc surface ($V_z \geqslant 0$) which ultimately results into a central plume. Thus, when buoyancy and rotation are simultaneously present, as above a heated rotating disc – the subject-matter of the present article -, there is a non-linear interaction of