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Thermal tomography utilizing truncated Fourier series approximation of the heat diffusion equation

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ABSTRACT

In a thermal tomography measurement setup, a physical body is sequentially heated at different source locations and temperature evolutions are measured at several measurement locations on the surface of the body. Based on these transient measurements, the thermal conductivity, the volumetric heat capacity and the surface heat transfer coefficient of the body are estimated as spatially distributed parameters, typically by minimizing a modified data misfit functional between the measured data and the data computed with the estimated thermal parameters. In thermal tomography, heat transfer is modeled with the time-dependent heat diffusion equation for which direct time domain solving is computationally expensive. In this paper, the computations of thermal tomography are sped up by utilizing a truncated Fourier series approximation approach. In this approach, a frequency domain equivalent of the time domain heat diffusion equation is solved at multiple frequencies and the solutions are used to obtain a truncated Fourier series approximation for the solution and the Jacobian of the time domain heat transfer problem. The feasibility of the approximation is tested with simulated and experimental measurement data. When compared to a previously used time domain approach, it is shown to lead to a significant reduction of computation time in image reconstruction with no significant loss of reconstruction accuracy.
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1. Introduction

Thermal tomography is an emerging non-destructive imaging technique which aims at recovery of three-dimensional images of the thermal conductivity and heat capacity of a physical body from non-invasive temperature measurements made on the surface of the body [1–11]. In the measurement process, the body is heated at a source location and temperature evolutions are measured at multiple measurement locations on the surface. The same process is then repeated for a number of source locations. Finally the measured temperature evolutions are used to estimate the unknown thermal conductivity and heat capacity as spatially distributed parameters which can be visualized as three-dimensional images. Potential applications of thermal tomography include characterization of thermal properties and non-destructive testing, such as detecting and locating air bubbles, cracks, porosity and other defects that alter thermal properties of materials.

The heatings and the temperature measurements can be chosen to be contact or non-contact based depending on the application. When physical contact with the body is practical, it is possible to use contact heaters, such as heating resistors, and contact temperature sensors, such as thermocouples or thermistors. Alternatively, if no contact is desired, inductive or laser heatings as well as thermography (IR-camera) measurements can be used.

Solving the thermal conductivity and the heat capacity of the body as spatially distributed parameters given the boundary measurements of heat transfer is a non-linear and ill-posed inverse boundary value problem which is unstable with respect to measurement and modeling errors. In this paper, this inverse problem is considered in the framework of Bayesian inversion [12,13].

Thermal tomography has been studied with simulations in [1–10]. The spatially distributed thermal conductivity of a two-dimensional (2D) body was estimated using simulated steady state measurements in [1] and that of three-dimensional (3D) bodies with known heat capacities using simulated transient state measurements in [2,3]. The estimation of either the spatially distributed thermal conductivity or the spatially distributed heat

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capacity of a 2D body while the other parameter was assumed known was studied in [4]. In [5], defects in 2D bodies were located using discrete variable thermal tomography, an approach where each pixel of the resulting image is allowed one of two possible sets of thermal properties, that is, the *a priori* known values of the intact or the defect area. Furthermore, 2D spatially distributed and time dependent thermal conductivity was estimated in [6] while the heat capacity was assumed known. The thermal conductivity and the shape of an inclusion were estimated in [7] using simulated steady state measurements with the thermal properties of the body otherwise known. More complexity was added in [8,9] when both the thermal conductivity and the heat capacity of 2D bodies were treated as unknowns and were estimated from transient boundary measurements.

In [9], it was demonstrated using simulated data from a 2D body that simultaneous estimation of spatially distributed thermal conductivity and volumetric heat capacity from transient boundary data is feasible when the boundary heat flux between the body and the surrounding medium is known. However, in practice such a measurement setup would not always be feasible as it requires the body to be insulated from the surrounding medium. In [10], the computational methods of [9] were extended towards a more practical setup of imaging bodies where the boundary heat flux between the body and the surrounding medium is not known (i.e. the body is not insulated). This was implemented by treating the surface heat transfer coefficient as a spatially distributed parameter at the surface, and estimating it simultaneously with the spatially distributed thermal conductivity and volumetric heat capacity.

In [11], the feasibility of thermal tomography was tested using experimental measurement data with computational methods modified from those of [10]. The data was measured from a mortar body using a prototype thermal tomography measurement device which uses heater resistors for heating and thermistors for temperature measurements at the surface of the body. The shape and location of an air cavity were clearly visible in the estimates of thermal conductivity and heat capacity, implying that thermal tomography with experimental measurement data is feasible.

We note that infrared thermography techniques [14–23] are somewhat related to thermal tomography, the main difference being that infrared thermography techniques are designed to detect defects that are located relatively close to the surface of the body, whereas thermal tomography aims at locating defects within the whole volume of the body and at giving quantitative solutions of the thermal conductivity and the heat capacity as well.

The forward model of thermal tomography which is used to model the time evolution of temperature inside the body is the time-dependent heat diffusion equation. In [11], the computational approach to solve the semi-discrete finite element approximation of this model and the related Jacobians was based on an implicit Euler scheme which makes estimating the thermal parameters time consuming. In this paper, we propose a truncated Fourier series approximation approach to reduce the computational cost of thermal tomography. In the proposed approach, the time-domain forward solution and Jacobians are approximated by a truncated Fourier series which is based on a small number of solutions of the frequency domain heat diffusion equation. The feasibility of the approach is evaluated with simulated and experimental measurement data by comparing the forward model solutions and the estimates of the thermal parameters to those obtained with the time domain approach of [11]. Previously, a similar Fourier series approximation for the solution of a time-dependent partial differential equation has been utilized for the solution of the time domain radiative transfer equation in [24]. In [25], a parallelized Fourier series truncated diffusion approximation was used to accelerate diffuse fluorescence tomography.

The rest of the paper is organized as follows. The modeling of heat transfer is discussed in Section 2 and the numerical implementation of the heat transfer modeling in Section 3. The estimation of the thermal parameters is discussed in Section 4. The measurement setup and parameter choices are discussed in Section 5 and the results using simulated and experimental measurement data are given and discussed in Section 6. Section 7 gives the conclusions.

2. Modeling of heat transfer

2.1. Time domain heat transfer

Let $\Omega \subset \mathbb{R}^3$ model the domain of the target, i.e. the domain of the body under investigation, with boundary $\partial\Omega$, let $\Xi_k \subset \partial\Omega$ ($k = 1, \ldots, N_\Xi$) be the surface patches that are covered by the heater elements and $\xi_j \in \partial\Omega$ ($j = 1, \ldots, N_\xi$) denote the locations of the point-like temperature sensors. In the measurement process, one of the heaters is turned on at a time for a time period t_{heat} and it produces a heat flux into the target at Ξ_k. This is followed by a cooling period of t_{cool} seconds before the next heater is turned on. While this is repeated for all heaters, the evolution of temperature is measured every Δt_m at all of the measurement locations for the duration of the measurement process $t_{\text{meas}} = N_\Xi(t_{\text{heat}} + t_{\text{cool}})$.

Heat transfer is modeled with the heat diffusion equation and the boundary conditions

$$c(x)\frac{\partial T(x,\ t)}{\partial t} = \nabla \cdot (\kappa(x)\nabla T(x,\ t)), \quad x \in \Omega \tag{1}$$

$$\kappa(x)\frac{\partial T(x,\ t)}{\partial \hat{n}} = q(x,\ t), \quad x \in \partial\Omega \tag{2}$$

$$T(x,\ 0) = T_0 \tag{3}$$

where $c(x)$ is the volumetric heat capacity, $\kappa(x)$ is the thermal conductivity, $T(x,\ t)$ is the temperature, $q(x,\ t)$ is the heat flux, x is the position vector in Ω, t is time, $\hat{n}$ is the outward pointing unit normal of $\partial\Omega$ and T_0 is the initial temperature of the target [26].

The boundary condition modeling the heat flux at the surface of the target can be split into heat flux between the heaters and the target, and heat flux between the target and the surrounding medium. Thus

$$q(x,\ t) = \begin{cases} b\big(T_{\text{H},k}(t) - T(x,\ t)\big), & x \in \Xi_k \\ h(x)(T_\infty(t) - T(x,\ t)), & x \in \partial\Omega_{\text{S}} \end{cases} \tag{4}$$

where $b = \kappa_b/L_b$ is the thermal contact conductance coefficient where κ_b and L_b are the thermal conductivity and the thickness of the contact layer between the heater and the target, $T_{\text{H},k}(t)$ is the temperature of the heater at the surface patch Ξ_k, $h(x)$ is the surface heat transfer coefficient, $T_\infty(t)$ is the temperature of the surrounding medium and $\partial\Omega_{\text{S}} = \partial\Omega \setminus \Xi_{k=1,\ldots,N_\Xi}$ is the part of the boundary that is not covered by heater elements [11].

2.2. Truncated Fourier series approximation

By using the notation $\tau(x,\ t) = T(x,\ t) - T_0$ and the Fourier transformation, the parabolic heat transfer problem (1)–(3) can be transformed into the elliptic frequency domain heat transfer problem

$$i\omega_j c(x)\tau(x,\ \omega_j) = \nabla \cdot (\kappa(x)\nabla\tau(x,\ \omega_j)), \quad x \in \Omega \tag{5}$$

with the boundary condition

$$\kappa(x)\frac{\partial\tau(x,\ \omega_j)}{\partial\hat{n}} = \begin{cases} b\big(\tau_{\text{H},k}(\omega_j) - \tau(x,\ \omega_j)\big), & x \in \Xi_k \\ h(x)\big(\tau_\infty(\omega_j) - \tau(x,\ \omega_j)\big), & x \in \partial\Omega_{\text{S}} \end{cases} \tag{6}$$

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