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A mathematical model of fluid flow in tight porous media based on fractal assumptions ☆

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ABSTRACT

Natural tight reservoirs are networks with high connectivity but low porosity, in which fractal behaviors have been widely observed and proven to affect the transport property significantly. The objective of this study is to establish a mathematical model to describe fluid flow in fractal tight porous media. To address this problem, four fractal dimensions were used: the pore size fractal dimension D_f, geometrical and hydraulic tortuosity fractal dimensions D_{τ_g} and D_τ, and the fractal dimension D_λ characterizing the hydraulic diameter-number distribution. The relationship among these fractal dimensions was analyzed and D_f was found equal to $D_\lambda + D_{\tau_g}$. Then a unified model connecting the porosity and D_f is deduced for arbitrary fractal tight porous media. Based on the scaling-invariant behaviors assumed, a fractal mathematical model is developed for the permeability estimation, which is fabricated only by fundamental and well-defined physical properties of D_f, D_τ, the scaling lacunarity P_λ, the range of the pore sizes, and the porosity of the fractal generator φ_0. To validate the permeability model, we developed an algorithm to model fractal tight porous media according to the scaling-invariant topography of fractal objects based on Voronoi tessellations, and to simulate fluid flow in these complex networks by Lattice Boltzmann method (LBM) at pore scale. Numerical experiments indicate that the hydraulic tortuosity fractal dimension D_τ is approximately equal to 1.1. Consequently, the fractal mathematical model was quantitatively determined and its performance was verified by the LBM simulations. Finally, the fractal mathematical model was rearranged into a permeability-porosity form for practical applications.

1. Introduction

Estimation of permeability is of pivotal importance for the description of different physical processes, such as oil recovery from underground reservoirs, coal-bed methane extraction from coal seams, generation of steam from geothermal reservoirs and contaminant migration in soils.

The natural porous media, such as geological oil/gas reservoirs, are always tight and consist of networks possessing low porosity but high connectivity. Due to the cascade effects of abundant external events together with internal processes, the microstructure might be disordered and complicated, with the size of pores/particle scaling-invariantly distributed and spanning several orders of magnitude [1–4]. In this study, we would like to approximate such media in nature by networks composed of intersecting capillaries with hydraulic diameters fractally distributed [5] and accounting for the effects on the permeability from possible scales.

In a reservoir evaluation, one of the main problems is to establish a relationship between the permeability and some fundamental physical properties, which can be easily measured and quantified for practical applications. One of the most well-known relationships was developed by Kozeny [6] and then modified by Carman [7] by simplifying the porous medium into a bundle of capillaries of equal length and constant cross section. The Kozeny-Carman (KC) formula relates the effective permeability (κ) to four structural parameters: the porosity (φ), the specific surface area (S), the hydraulic tortuosity (τ), and the shape factor (g). The product $g\tau^2$ is known as the Kozeny constant.

However, the KC formula is a semi-empirical permeability-porosity relationship and might fail to work in practice. Substantial differences have been observed between measured permeabilities,

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even when the porous media share the same statistical quantity of fundamental physical properties. More careful investigations have shown that the hydraulic tortuosity cannot be a function of porosity only [8–11], and the Kozeny constant will alter with pore structures [12–15]. Thus, except for a detailed understanding of the geometries of a porous medium, the determination of an appropriate relationship is hard because of the large number of related parameters [5].

The permeability investigation of porous media might fall into one of three categories: experiment-based analysis, analytical derivations, and numerical simulations. The experimental studies are always based on natural media, such as rock cores. The rock cores are useful for characterizing individual formations, but the microstructural effects on fluid flows may be ignored due to the continuum assumption of a porous sample [5,16]. Thus, semi-empirical parameters must be introduced as responsible for the unclear effects, which makes the experimental results of limited use in understanding fundamental mechanisms of fluid flow [11,15,17]. Furthermore, it is impossible to independently manipulate properties like porosity, pore size distribution, and connectivity within natural porous media, especially for tight reservoirs.

In an analytical approach, a mathematical framework could be established for a problem with clear physical meanings, which can guide us to adapt the inherent coefficients to suit the experiments [14]. Generally, the porous media will be simplified into conceptual models sharing some geometrical features with the natural ones. There are two kinds of models to mimic the fractal behaviors of natural porous media: the pore fractal and solid fractal models. Following the solid fractal scheme, Pitchumani and Ramakrishnan [18] proposed a permeability model by assuming natural reservoirs as fractal capillaries, different from the idealized arrangements of tube bundles with the same length. Subsequently, Yu and Cheng [19] analytically deduced a fractal permeability model based on the assumption that the size distribution of the capillary hydraulic diameter follows fractal statistics as well as the scaling/tortuosity law between the hydraulic diameter and the tortuosity proposed by Wheatcraft and Tyler [20]. Thereafter, a lot of investigations were conducted on the transport properties of fractal porous media [16,21,22]. These researches shed light on the subject of establishing permeability-porosity relationship for fractal porous media; however, it was not consistent with practical situations since it assumed no intersection of the capillary tubes.

Due to the intersected nature of capillaries in natural porous media, networks or bead-based models have been widely used to investigate the transport properties. For porous media with fractal behavior, the basic idea is to reformulate the classical KC equation by replacing the physical properties determined solely by the geometry of the medium. Among the few reviewed here, Costa [5] proposed a two-parameter permeability-porosity equation, and then Henderson et al. [9] analytically derived a three-parameter permeability model. Based on the regular pore fractal models (bead-based approach), Wang et al. [11] analytically derived a semi-empirical fractal permeability model, indicating that the porosity to the power of $(4 - D_f)/(2 - D_f)$ is proportional to the permeability of a two-dimensional porous medium, with D_f as the pore fractal dimension. Jin et al. [15] investigated the effect of fractal pore structure on the permeability, hydraulic tortuosity, as well as the KC constant. Direct investigations are more reliable than those assuming that the capillary tubes are apart from each other. Most importantly, the networks or bead-based models easily allow for almost any periodic, random, or fractal geometry [23], as well as recreations of rock geometry from imaging data [24,25].

For the complex hydrodynamic problems, more and more efforts are now devoted to direct numerical simulations at pore scale. By such solutions, relevant parameters could be considered or neglected independently, and the geometrical effects could be evaluated with an uncertainty as small as possible compared with the coupling results from experiment-based methods [26]. Moreover, the computational fluid dynamics (CFD) models are not limited to any experimental techniques, scales, or environments [11]. However, different relationships can be derived from the same numerical results without a guiding model behind it. Thus, the mathematical framework should be established in advance, and then the pore-scale modeling and micromodel experiments should be used to mechanically understand the basic physics [15].

In addition, the unmet need in studying the permeability of geological porous media is that the conceptual models should share similar geometries or physical properties with the natural ones. Conceptual models based on regularly or stochastically arranged cylinders or bead-based approaches are simple and good choices to reconstruct the natural porous structures. Recently, Naraghi and Javadpour [27] investigated the transport properties of the shale-gas systems. In their study, the pore network in shale reservoirs was represented by a stochastic bed-packing model. Landry et al. [28] reconstructed complex nanoporous media with fractal behaviors by the bead-based approaches, and investigated the apparent permeability value by direct simulations. However, the porous media constructed by the approaches above are limited by the packing density of the shapes to high porosities (>0.2) [29]. Actually, many hydrocarbon bearing sedimentary rocks such as tight gas sand and shale, as well as coal seams, due to severe consolidation, have very low porosity (<0.1), but are still highly connected. Recently, based on two-dimensional Voronoi diagrams, Wu et al. [30] developed an algorithm for generating microfluidic low porosity random porous media for studying single and multiphase flow. However, as noted before, the sizes of the hydraulic diameters might be fractally distributed in natural reservoirs [5,19], and the scaling effects on the permeability are significant [9,11,5,16]. Thus, in fractal porous media modeling, the low porosity, randomness, and high connectivity should all be taken into account, as well as the fractal behavior of the diameter distribution due to the possible effects from different scales.

In this paper, a general Hagen-Poiseuille (HP) equation and the fractal assumptions (including the scaling/tortuosity characteristics and fractal property of hydraulic diameters) are used to derive a permeability model for a kind of intersecting, fractal, and tight porous media. For convenience, hereafter we will call this kind of medium *fractal tight porous medium*. To validate our proposed model, we developed a modeling approach to generate fractal porous media possessing low porosity, high connectivity, and random characteristics. Based on such fractal porous models and by Lattice Boltzmann simulations at pore scale, we investigated fluid flow through a tight fractal porous medium. The validation of our proposed fractal permeability model and that of the modeling approach are both verified.

2. Tortuous HP equation

Let $A_{\parallel}$ and A denote the cross-sectional area of the capillary perpendicular to the direction of the macro flow and the local direction of fluid flow, respectively.

For a capillary with noncircular cross-sectional shape, its hydraulic diameter λ is defined to be 4 times the cross-sectional area A divided by the perimeter of fluid confined boundary P_r:

$$\lambda = \frac{4A}{P_r}. \tag{1}$$

Assuming, for simplicity, that all capillary tubes possess the same cross-sectional shape, the relationship between λ and the cross-sectional area A can be expressed by introducing a geometric dependent parameter g, and reads: