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Theoretical accuracy of anisotropic thermal conductivity determined by transient plane source method

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ABSTRACT

The transient plane source (TPS) method could measure the in-plane thermal conductivity and through-plane thermal conductivity of anisotropic materials through one single test once the volumetric heat capacity is known. The practical thicknesses of the heating element and the insulation layer deviate from the plane source assumption and have an influence on the accuracy of the isotropic thermal conductivity of bulk specimen and film specimen determined experimentally. The theoretical accuracy of measured anisotropic thermal conductivities will also be affected by the practical sensor thickness. A numerical study is conducted to investigate the deviation of anisotropic thermal conductivity due to the non-compliance of the theoretical assumption of TPS method. The influence of the practical sensor thickness on the theoretical accuracy of different thermal conductivities and different anisotropic ratios is discussed. The simulation studies show that the deviation brought by the plane source assumption, i.e., with zero thickness, becomes significant for materials with high thermal conductivity (thermal diffusivity) and can be improved by employing sensor with larger radius.

1. Introduction

TPS method was proposed by Gustasson [1] and has been developed to measure isotropic thermal conductivity and thermal diffusivity of solid, powder, porous material and liquid. TPS method could measure the isotropic thermal property of bulk specimen, slab specimen and film specimen. TPS method can also be used to measure anisotropic thermal conductivity [2–6] and specific heat capacity [7] and became an ISO standard in 2008 [8].

Anisotropic materials are widely used in mechanics, electronics, chemical industry and aerospace. It is well known that materials like wood, paper, clothes, laminated material, reinforced fibers composite material, woven fabric composite material, crystalline material, nano-film, nano-wire and nano-tube, often have strong direction-dependent properties [9,10]. To understand the heat conduction behavior of anisotropic materials, it is essential to obtain the anisotropic thermal conductivity. However, it is a much difficult and rather lengthy process to determine the anisotropic thermal conductivity compared with the determination of isotropic thermal conductivity.

Most widely used methods of measuring thermal conductivity of isotropic materials, such as steady state method, laser flash method [11], hot wire/strip method [10,12] and 3 ω method [13], can be extended to measure the anisotropic thermal conductivity by changing the test direction of sample. The TPS method could applied to anisotropic materials, in which the thermal properties along two of the orthogonal and principal axes are the same (in-plane thermal conductivity) but are different from those along the third axis (through-plane thermal conductivity), with known volumetric heat capacity [2,5,6,8,14–17]. Compared with the other methods, TPS method has many special advantages. For example, it could measure the through-plane thermal conductivity and in-plane thermal conductivity through one single test once the volumetric heat capacity is known. In addition, TPS method only requires small samples and it is especially suitable for materials with limited sample size. The steady state method cannot be used to measure the anisotropic thermal conductivity of some woven fabric materials because it requires large size samples of 3 dimensional directions but the manufacturing technology is not allowed. The radius of fiber bundle or the pore size of typical woven fabric materials is in millimeter range which is much larger than the diameter of hot wire or the thickness of hot strip. Therefore, the hot wire/strip method is also not suitable under such condition. The laser flash method is a transient method to determine the

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Nomenclature

a	thermal diffusivity, mm^2/s
a_x	in-plane thermal diffusivity, mm^2/s
a_z	through-plane thermal diffusivity, mm^2/s
c	specific heat capacity, J/kg K
C	heat capacity per volume, J/m^3 K
$D(\tau)$	dimensionless time function
h	thickness of slab, mm
I_0	modified zero order Bessel function
k, l	intermediate variable
m	number of the concentric rings
P_0	heating power of the sensor, W
Δp_{prob}	probing depth $\Delta p_{prob} = 2\sqrt{at}$, mm
R_0	initial resistance of the sensor before heating, Ω
$R(t)$	resistance of the sensor at time t, Ω
r	radius of outer ring of the sensor, mm
ΔT	temperature increase of the sensor, K
ΔT_i	temperature difference across the heat sensor insulation layer, K
$\Delta T(\tau)$	the temperature increase of the sensor outer surface, K
t	heating time, s

Greek symbols

α	temperature coefficient of the resistance, 1/K
δ	sensor insulation thickness, mm
λ	thermal conductivity, W/m K
λ_{xy}, λ_x, λ_r	in-plane thermal conductivity, W/m K
λ_z	through-plane thermal conductivity, W/m K
ρ	density, kg/m^3
τ	dimensionless time given by $\tau = \sqrt{t/\Theta}$
τ_x	dimensionless time given by $\tau = \sqrt{t/\Theta_x}$
Φ	internal heat source, W/m^3
Θ	characteristic time, $\Theta = r^2/a$, s
Θ_x	characteristic time, $\Theta_x = r^2/a_x$, s

thermal diffusivity and it is not suitable for measuring materials with macro pores which are transparent to laser flash.

Uncertainty analysis is an important part for any measurement method. According to Malinarič's classfication [18], the uncertainty of dynamic measurement mainly comes from the parameters (including geometric and operational) determination, evaluation method and the theoretical assumptions. For TPS method, the uncertainty in determination of geometric parameters (such as sensor radius, sensor thickness, insulation thickness) and operating parameters (such as temperature, temperature coefficient of resistance, heating power and heating time) may cause errors. The least square fitting procedure of determining the thermal properties will also brought in deviation. Many works have been conducted to analyze the influence of parameters determination and data reduction method on the accuracy of dynamic tests such as TPS method [18–22]. Recently, more attentions have been focused on the influence of the theoretical assumptions and in this regard numerical simulation is a very efficient and convenient tool [7,23–26]. Coquard et al. [23] analyzed the accuracy of TPS method when applied to low density insulating materials using simulation. They found that the accuracy of TPS method becomes questionable when measuring high insulating materials because of their thermal inertia are two orders of magnitude lower than that of the sensor. At the same time, we developed a numerical simulation algorithm to study the influence of the mathematical assumptions deviating from the practical measurement on the theoretical accuracy of the bulk thermal conductivity estimated [24]. In this literature, the influence of sensor thickness on measuring isotropic materials with low thermal conductivity was mainly concerned. Thick mica sensor should be adopted when measuring thermal conductivity at elevated temperature. However, the accuracy of TPS method is indeed a big problem when measuring extremely low thermal conductivity materials with thick mica sensor because the heat dissipates from the sensor side cannot be neglected. Similar numerical simulation works are also conducted to reveal the theoretical accuracy of TPS method when measuring thermal conductivity of thin film materials [25]. Warzoha et al. studied the effect of natural convection on the thermal conductivity of liquids measured by TPS method using numerical simulation and proposed a correlation to predict the onset of natural convection [26].

Although the nominal test range of TPS method can be as high as 500 W/m K, to the best of our knowledge there has no relevant accuracy analysis of measuring high thermal conductivity materials and anisotropic materials using TPS method in open literature. In this study, the theoretical accuracy of anisotropic thermal conductivity determined by TPS method is numerically investigated for different materials with considering the practical sensor thickness. The research method is similar to that of literatures [24,25], but this study mainly focus on anisotropic materials.

2. Theoretical basis of TPS method

TPS method could measure the in-plane thermal conductivity (λ_{xy}) and through-plane thermal conductivity (λ_z) of anisotropic materials if the main directions of the material are orthogonal (denoted as x, y and z axis) and thermal properties in the x-y plane are uniform ($\lambda_x = \lambda_y = \lambda_{xy}$). When perform the thermal conductivity measurement, a sensor (Fig. 1a) is placed between two pieces of anisotropic samples with smooth surface to form a sandwich structure. The heating sensor shown in Fig. 1b has a double spiral shape of nickel and it is covered by two thin pieces of kapton or mica insulation layer to keep electric insulation and a high structure mechanical strength.

TPS method is based on following assumptions:

(1) The bifilar probe can be approximated by a number of concentric and equally spaced circular line sources.
(2) The dimension of the test material is semi-infinite. It can be guaranteed by performing the test with test time and sample size by satisfying some criteria which will be introduced later in this section.
(3) The thickness of the sensor insulation layer and the heat element can be neglected.

The descriptions and equations presented below are based on the above three assumptions. The third assumption is not appropriate when measuring extremely low isotropic thermal conductivity materials with a thicker mica layer coated sensor. The uncertainty brought by this assumption when determining anisotropic thermal conductivity will be revealed in this work using "numerical experiment".

In the test, the sensor is used both as a heat source and as a temperature sensor. When the sensor is heated by a constant electric power, the temperature of sensor will increase with time and it can be obtained from the variation of its electric resistance when the temperature coefficient of resistance (α, 1/K) of the heat ele-