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A dynamic subgrid-scale modeling framework for Boussinesq turbulence

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ABSTRACT

A modular dynamic subgrid-scale modeling framework is presented for large eddy simulation of two-dimensional Boussinesq turbulence. The procedure we put forth in this study allows us to couple the structural subgrid-scale parameterization models with the functional models by minimizing the error between them. In particular, the approximate deconvolution procedure is used to estimate the Smagorinsky and Baldwin-Lomax eddy viscosity constants and the associated turbulent Prandtl numbers self-adaptively from the resolved flow quantities. Our numerical assessments for solving the Rayleigh-Bénard turbulent thermal convection problem show that the proposed approach could be used as a viable tool to address the turbulence closure problem for the Boussinesq setting due to its accuracy and flexibility.

1. Introduction

Turbulence is a flow phenomenon characterized by chaotic motions represented by large variations in the velocity field in the domain of interest combined with high momentum transfer through advection. Convective motion is generated by spatial variations in an advected scalar field (in our case temperature) which in turn transports the scalar itself. As such, this physical process is of exceeding importance in a large number of fields ranging from engineering applications to geophysical and astrophysical flows. A classical problem in non-isothermal flow is the Rayleigh-Bénard convection problem pertaining to a buoyancy driven flow in a fluid layer heated from below which has been extensively used to understand the transport properties of convective flows and considerable research is as yet underway both from a theoretical as well as a computational and experimental point of view [1–3]. Investigations related to scaling laws are important for determining convection dominated heat transfer physics, particularly for emerging nanofluidic applications. The reader may refer to the works in [4–13] where a wide variety of numerical methods for different governing equations are implemented.

It is thus natural that a considerable amount of attention is given to the computational study of the Rayleigh-Bénard problem due to the potential implications of an improved understanding of its convective behavior. Modeling the behavior of the temperature gradient driven flow (particularly at higher values of the controlling parameter, the Rayleigh number) requires a difficult decision with respect to the choice for a solution methodology. A natural choice for high fidelity simulation data would be through direct numerical simulation (DNS) of the Navier-Stokes (NS) equations. However it is seen that the scale separation inherent in turbulent flows makes the use of DNS prohibitively expensive for most problems of practical interest since that would require the capture of all the scales in the flow from the integral to the Kolmogorov length scales. To compensate for these immense computational and memory overhead requirements for DNS, researchers turn to turbulence models which aim to capture the behavior of the larger scales in the fluid through modeling the effect of the smaller structures on them. Turbulence modeling efforts can be categorized in two main directions: the Reynolds averaged NS (RANS) based models where a turbulence model is applied for all the relevant scales of the flow to determine the aggregate behavior of the physics and large eddy simulation (LES) based approximations where only the smaller features of the flow are modeled whereas the most energetic large scales are resolved. The RANS approach does not resolve any scales of turbulence but only models the turbulence spectrum as it is based on the temporal averaging of NS equation. However, LES models are able to resolve and capture the influence of larger length-scales of turbulence while modeling only the influence of smaller scales (unresolved subgrid scale as determined by the computational mesh size). By removing the high wavenumber content of the flow through a low-pass filtering procedure, LES has been proven to be an accurate and computationally feasible approach for calculations of complex turbulent flows [14–17]. Once filtered, the governing equations need to be closed due to their nonlinearity [18,19].

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Nomenclature

Ra	Rayleigh number ($Ra = \frac{\alpha g \Delta T h^3}{\nu\kappa}$)	C_B	Baldwin-Lomax model coefficient
Pr	Prandtl number ($Pr = \frac{\nu}{\kappa}$)	δ	characteristic grid filter scale
Re	Reynolds number ($Re = \sqrt{Ra/Pr}$)	$\tilde{\delta}$	test filter scale
Nu	Nusselt number	$\vert S\vert$	absolute strain rate tensor
θ	dimensionless temperature	l_0	mixing length scale
ω	dimensionless vorticity	β	relaxation parameter in AD
ψ	dimensionless streamfunction	N	the order of Van Cittert iterations
$\boldsymbol{u}$	dimensionless velocity vector	N_x	numerical resolution in x-direction
u	dimensionless velocity x-component	N_y	numerical resolution in y-direction
v	dimensionless velocity y-component	ΔT	reference temperature difference
ν_e	eddy viscosity	α	thermal expansion coefficient
Pr_t	turbulent Prandlt number	ν	fluid kinematic viscosity
S_θ	SGS term for temperature equation	κ	thermal diffusivity
S_ω	SGS term for vorticity equation	h	height of the channel
C_S	Smagorinsky model coefficient	w	width of the channel

LES closures have been used extensively over the last few decades in modeling advection dominated flows and several subgrid modeling approaches have been developed for a wide variety of flows [20–32]. The basic methodology for developing an LES governing equation is the application of a low-pass spatial filter to the NS equations which removes the resolution requirement of small-scale turbulence followed by the treatment of the closure problem due to the presence of the nonlinear advective terms in the governing equations. This leads to a significant reduction in computational expense as much coarser meshes are now used. The end aim of closure modeling is to obtain an accurate representation for energy and momentum transfer mechanisms for simulated flows. The classical Kolmogorov cascade theory for fully turbulent flow which defines a nonlinear interaction of energy cascading from the larger eddies in the flow to the smallest scales where the energy is dissipated by viscosity in the form of heat [33]. However, a unified LES closure scheme is a challenging task due to the different energy transfer characteristics (e.g., forward enstrophy and inverse energy cascades in quasi two-dimensional geophysical flows where stratification and rotation suppress vertical motions in the thin layers of fluid).

Two major schools of thought can be discerned in the general body of LES closure research with one adhering to the use of predefined subgrid-scale (SGS) models to approximate the effect of the subgrid scale structures after the use of an explicit filtering procedure, and the other committed to total dissipation of subgrid scale stresses through the use of an implicit filter incorporated in a spatially biased numerical scheme. While arguments have been made in the favor of the former due to its more physical approach to the closure problem, the lower computational expense of the latter has also made implicit LES (ILES) attractive for more practical flows. Within the general body of SGS models with explicit filtering methodologies lie two approaches to the closure problem called the structural and functional approaches. An important observation here is that both functional and structural closure models and their variants are considered explicit LES closures due to the fact that the underlying equations are modified before discretization. The simplest form of LES is just to increase the viscosity until the viscous scales are resolved by underlying computational mesh. This added viscosity is generally called the *turbulent eddy viscosity* and becomes the foundation of the main stream turbulent closure models. Eddy viscosity models are consistent with Kolmogorov's ideas about the energy spectrum of three-dimensional isotropic turbulence where energy is injected into the flow at large scales and is gradually transferred by nonlinear cascading processes to smaller and smaller scales until it is dissipated near the viscous dissipation scale. Therefore the first and ultimately simplest approach to parameterize these *eddy interactions* is to use functional models.

The functional SGS closure modeling approach pertains to the use of a user defined eddy viscosity or a heuristic for the calculation of eddy viscosity at different scales. One of the most celebrated functional closures include the Smagorinsky model [20] and its dynamic version [23,25] which have been applied successfully to a large number of practical flows [16,17]. Both these versions incorporate heuristics from the famous mixing length theory to define an artificial dissipation in the velocity field as a function of the local (or spatially averaged) gradients. The heuristic culminates with the user-defined choice of a parameter (known as the Smagorinsky constant) which is fixed at a constant value in the regular case and updated during the computation of the flow field in the dynamic case. It must be mentioned here that although we determine the LES equations using a low-pass filtering procedure, a filter is not specified explicitly. The dynamic Smagorinsky model has been used with relative success in many different fields in comparison to the standard Smagorinsky model due to the observation of a wide spread of Smagorinsky constants in literature [34–38]. This mixing length based heuristic for additional dissipation may also be devised for use in two-dimensional turbulence through the use of an artificial dissipation determined from the local vorticity field also known as the Baldwin-Lomax model. The Baldwin-Lomax model is better suited to the concept of an enstrophy cascade from the larger to the smaller scales (as witnessed in two-dimensional turbulence).

The structural approach to obtaining a turbulence closure lies in the prescription of a closed system of equations without the use of any artificial dissipation through the introduction of a hypothetical eddy viscosity. A popular structural closure for subfilter scale (SFS) modeling is the approximate deconvolution (AD) methodology [28] which has been studied in considerable theoretical detail [39–45] and has also been applied to a large number of practical flows [46–53]. AD-LES was conceptually developed from the image processing community for the reconstruction of sub-filter scales using Van-Cittert iterations which is basically an iterative substitution methodology involving repeated filtering to recover an approximation for the unfiltered quantity from the filtered variable [54–56]. The primary strength of this structural SFS modeling approach is the lack of any physical assumptions or any phenomenological arguments about the flow which makes the method particularly appealing for flows with an inverse energy cascade as is the case in two-dimensional turbulence [57,58].