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Numerical investigation of mixed convection of Bingham fluids in cylindrical enclosures with heated rotating top wall

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ABSTRACT

The steady-state laminar mixed convection of yield stress fluids obeying the Bingham model in a cylindrical enclosure with a heated rotating top cover has been numerically analysed based on axisymmetric incompressible flow simulations. Yield stress effects on heat and momentum transport have been investigated for an aspect ratio (height/radius) of unity (*i.e.* $AR = 1$) for a range of different values of nominal Prandtl, Richardson and Reynolds numbers given by $10 \leqslant Pr \leqslant 500$, $0 \leqslant Ri \leqslant 1$ and $100 \leqslant Re \leqslant 3000$. The mean Nusselt number $\overline{Nu}$ has been found to decrease sharply with increasing Bingham number Bn, but subsequently $\overline{Nu}$ approaches asymptotically to a value of unity, which is indicative of conduction-driven transport. It has also been found that $\overline{Nu}$ increases with increasing values of Prandtl and Reynolds numbers for both Newtonian (*i.e.* $Bn = 0$) and Bingham fluids. In contrast, $\overline{Nu}$ decreases with increasing Ri for both Newtonian and Bingham fluids for small values of Bingham number, whereas $\overline{Nu}$ remains insensitive to the variation of Ri for large values of Bingham number. The variation of torque coefficient C_T, which gives a quantitative measure of power consumption, has also been investigated. The torque coefficient C_T has been found to increase with increasing Bn whereas it decreases with increasing Re. It has also been found that C_T decreases slightly with increasing Ri for small values of Bn, whereas it becomes insensitive to the variation of Ri for large Bingham numbers. For the fully forced convection ($Ri = 0$) case, Pr does not have a significant influence on C_T. However, in the case of mixed convection C_T increases with increasing Pr. The simulation data has been used in conjunction with a detailed scaling analysis to propose a correlation for $\overline{Nu}$ for the range of Re, Ri and Pr considered here.

1. Introduction

Mixing and heat transfer play pivotal roles in several engineering applications relevant to chemical processing, bio-chemical synthesis, polymer processing, food preparation, modern agriculture and many other related industries. Augmentation of mixing and heat transfer in laminar flow conditions can be challenging but it is possible to enhance the rates of heat transfer and mixing by rotating one of the covers of a cylindrical container to produce a swirling flow. Although superficially it seems to be a simple process, the heat transfer rate in this configuration depends on many parameters, such as the container geometry and rotational speed of cover. Flow in cylindrical enclosures with a rotating cover has been extensively analysed owing to its wide-ranging roles in the aforementioned engineering applications. However, the majority of these analyses have been conducted only for Newtonian Fluids (where the viscous stress is directly proportional to strain rate). Vogel [1,2], Ronnenberg [3], and Bertela and Gori [4] pioneered the analysis of this configuration for Newtonian fluids. The findings of these studies [1–4] were confirmed and extended by Escudier [5] based on an experimental analysis where the stability criterion for vortex breakdown was identified in terms of aspect ratio (AR = height/radius = H/R) and Reynolds number ($\Omega R^2/\nu$). In addition to these experimental studies, several numerical investigations [6–10] involving Newtonian fluids have been carried out. Lee and Hyun [8] analysed the effects of Prandtl number on heat transfer rate in this configuration and revealed that Prandtl number has an important influence on the heat transfer characteristics and advective transport has been found to strengthen with increasing Prandtl number. Iwatsu [9] investigated the effects of Reynolds and Richardson numbers in the range of $100 \leqslant Re \leqslant 3000$ and $0 \leqslant Ri \leqslant 1$ for $Pr = 1$ on the flow pattern and heat transfer rate for swirling flows in cylindrical enclosures with an aspect ratio of unity (*i.e.* $AR = H/R = 1$) and a heated rotating top wall. The analysis by Iwatsu [9] revealed that advective (diffusive) transport

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Nomenclature

a, a_0 [-]	bridging function
a_1, a_2 [-]	correlation parameters
AR [-]	aspect ratio ($AR = H/R$)
b, b_0 [-]	bridging function
b_1, b_2 [-]	correlation parameters
Bn [-]	Bingham number
c_p [J/kg K]	specific heat at constant pressure
C_T [-]	torque coefficient
e_a [-]	relative error
f_1,f_2,f_3 [-]	functions
g [m/s^2]	gravitational acceleration
Gr [-]	Grashof number
h [W/m^2 K]	heat transfer coefficient
H [m]	height of cylindrical enclosure
k [W/m K]	thermal conductivity
k_0,k_1 [-]	correlation parameters
m_0,m_1 [-]	correlation parameters
Nu [-]	Nusselt number
$\overline{Nu}$ [-]	mean Nusselt number
p [-]	theoretical order of accuracy
Pr [-]	Prandtl number
q [W/m^2]	heat flux
R [m]	radius of cylindrical enclosure
Ra [-]	Rayleigh number
Re [-]	Reynolds number
Ri [-]	Richardson number
T [K]	temperature
u [m/s]	radial velocity component
U (m/s)	characteristic velocity scales in radial direction
v [m/s]	tangential velocity component
V (m/s)	characteristic velocity scales in tangential direction
V_ϕ [-]	non-dimensional swirl velocity ($V_\phi = vH/\alpha$)
w [m/s]	vertical (axial) velocity component
α [m^2/s]	thermal diffusivity
β [1/K]	coefficient of thermal expansion
$\dot{\gamma}$ [1/s]	shear rate
δ,δ_{th} [m]	hydrodynamic and thermal boundary layer thickness
θ [-]	non-dimensional temperature ($\theta = (T - T_C)/(T_H - T_C)$)
θ_1 [-]	parameter
μ [Ns/m^2]	plastic viscosity
μ_{yield} [Ns/m^2]	yield viscosity
v [m^2/s]	kinematic viscosity
ρ [kg/m^3]	density
τ [N/m^2]	shear stress
τ_y [N/m^2]	yield stress
Ω [1/s]	angular velocity
ψ [m^2/s]	stream function
Ψ [-]	non-dimensional stream function ($\Psi = \psi/\alpha$)

Subscripts

$Bn = 0$	Newtonian fluid case
C	cold wall
eff	effective value
H	hot wall
max	maximum value
r	radial direction
ref	reference value
$wall$	wall value
wf	condition of the fluid in contact with the wall
z	axial direction
ϕ	tangential direction

Special characters

ΔT [K]	difference between hot and cold wall temperature (= $(T_H - T_C)$)
$\Delta_{\text{min,cell}}$ [m]	minimum cell distance
r [-]	grid expansion ratio

strengthens (weakens) and, accordingly, the mean Nusselt number decreases with increasing Richardson number. Compared to Newtonian fluids, relatively limited efforts have been directed to the analysis of mixed convection of non-Newtonian fluids (where the viscous stress is not proportional to the strain rate). Although both water and air, being the most abundant fluids on the planet, are Newtonian fluids, and it is logical that most previous work concentrated on such Newtonian fluids, virtually all synthetic, food-based and biological fluids (*e.g.* detergents, toothpaste, cosmetics, ketchup, custard, blood) are non-Newtonian in character. Some electro-rheological/magneto-rheological fluids exhibit yield stress and behave only as fluids once a threshold stress is surpassed. It is possible to modulate the yield stress by using an electrical/magnetic field in electro-rheological/magneto-rheological fluids. Thus, from an engineering perspective, the knowledge of mixed convection of more rheologically complex fluids than water or air is essential.

One of the first investigations related to swirling flows for non-Newtonian fluids was performed for flows over a rotating disk, which is commonly referred to as the von-Karman flow [11–13]. Most analyses on the non-Newtonian fluid flow in cylindrical enclosures with a rotating cover were carried out for viscoelastic fluids [14,15]. Escudier and Cullen [14] experimentally analysed cylindrical enclosures with a rotating top cover for shear-thinning viscoelastic fluids, and reported that the vortex structure is different from the Newtonian fluid case and an intense toroidal vortex that drives a secondary low intensity vortex was produced in the vicinity of the rotating cover. Stokes and Boger [15] proposed a regime diagram for flow stability based on Reynolds and Elasticity numbers for viscoelastic fluids in cylindrical enclosures with a rotating cover. A recent experimental analysis of heat transfer under elastic turbulence of viscoelastic fluids within cylindrical enclosures with a rotating top cover was reported by Traore et al. [16]. It was found that the heat transfer rate under elastic turbulence might locally increase by up to four times as compared to the purely conduction regime. It was indicated by Traore et al. [16] that the elastic turbulence can augment the heat transport rate in situations where the inertial turbulence is difficult to obtain, such as in the case of microscopic flows.

Analysis of power-law fluids in cylindrical enclosures with a rotating cover was carried out by Böhme et al. [17]. The influence of shear-thinning on vortex breakdown (observed before by Vogel [1,2] and Escudier [5] for Newtonian fluids) in cylindrical enclosures with a rotating cover was experimentally and numerically analysed by Böhme et al. [17]. They constructed an aspect ratio - Reynolds number (AR - Re) diagram, representing the domain of vortex breakdown for shear-thinning fluids. A few recent analyses [18–20] have concentrated on rotating disk configuration for yield stress fluids. The mass transfer of yield stress fluids in such applications has recently been investigated by Rashaida et al. [18] and semi-analytical approaches for analysing swirling flow of Bingham fluids over a rotating disk has been reported by Ahmadpour and Sadegyhy [19] and Guha and Sengupta [20]. However, to date, there is no analysis in the existing literature that deals with flow