International Journal of Heat and Mass Transfer 109 (2017) 123–133

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Topology optimization of heat sinks in natural convection considering the effect of shape-dependent heat transfer coefficient

Younghwan Joo, Ikjin Lee, Sung Jin Kim *

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon 34141, Republic of Korea

ARTICLE INFO

Article history:
Received 26 October 2016
Received in revised form 25 January 2017
Accepted 25 January 2017
Available online 10 February 2017

Keywords:
Shape-dependent heat transfer coefficient
Topology optimization
Heat sink
Natural convection

ABSTRACT

Heat sinks in natural convection are thermally optimized by using the topology optimization method. To investigate the shape-dependent effect in natural convection, a new surrogate model accounting for the variation of the heat transfer coefficient within the computational domain is proposed. In order to validate the surrogate model, the result from topology optimization with the proposed surrogate model is compared to the plate-fin heat sink optimized using the existing correlation. From the comparison, it is found that the optimum channel spacing for the plate-fin heat sink is successfully reproduced by applying the surrogate model. With the validated surrogate model, a new conceptual design for a heat sink is obtained in the physical domain for which a conventional heat sink has been designed. To reflect the manufacturing constraints for mass production, guidelines for design simplification are suggested and applied to the conceptual design. Through the numerical simulation, it is found that the topology-optimized heat sink has 15% lower thermal resistance and 26% less material mass than the conventional heat sink.

1. Introduction

Natural convection heat sinks are widely used because of their simplicity and reliability [1,2]. In most cases, the size of the physical domain for designing heat sinks is limited. Therefore, studies on the natural convection heat sinks have mainly focused on obtaining the optimum fin geometries under a given physical domain. In conventional methods, fin shapes such as a plate-fin or a pin-fin are assumed a priori and geometric parameters subject to these fin shapes are optimized [3,4]. Recently, some researchers have applied a new method, called topology optimization, to thermal optimization of heat sink geometries without specifying a fin shape. Topology optimization has been commonly used in structural problems [5]. In this method, the computational domain is divided into a large number of elements. Each element has a relative density that determines whether the element is occupied by the solid material or not. Therefore, the distribution of the relative density within the computational domain determines heat sink geometries.

Some researchers have tried to apply the topology optimization method to designing a heat sink [6–8]. Dede et al. [6] suggested heat sink geometries in a 2-D computational domain. To consider natural convection heat transfer occurring at the surface of a heat sink, they employed a uniform value of the heat transfer coefficient at every surface element. However, applying the same value of the heat transfer coefficient to every surface element is not accurate because the heat transfer coefficient is significantly affected by the local shape of a fin structure. Iga et al. [7] employed the topology optimization method considering shape-dependent heat transfer coefficient for the case of forced convection. They proposed a surrogate model by which the heat transfer coefficient is predicted at solid-void interfacial elements. To construct the surrogate model, they investigated the variation of the heat transfer coefficient numerically using a variety of fin structures with sinusoidal profiles. Because the local shape of a structure is much more complex than the sinusoidal profiles, their surrogate model may not be generally applicable to arbitrary shapes encountered in topology optimization.

To overcome the difficulties associated with accurate prediction of the heat transfer coefficient distribution for flow over the arbitrary shapes in topology optimization, numerical investigations of full conjugate heat transfer problems have been attempted. In the 2-D computational domain, Yoon [9] and Alexandersen et al. [10] suggested optimal geometries for heat sinks in forced convection and natural convection, respectively. Under a given relative density field, they solved the combined conductive and convective heat transfer problem to obtain the velocity field and

---

* Corresponding author.
E-mail address: sungjinkim@kaist.ac.kr (S.J. Kim).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.01.099
0017-9310/

Nomenclature

A	surface area (m^2)
$\mathbf{B}$	derivative matrix (1/m)
b	internal heat generation per unit volume (W/m^3)
c	thermal compliance (K W)
c_p	specific heat (kJ/kg °C)
$\mathbf{D}$	thermal conductivity matrix (W/m K)
El	Elenbaas number (–)
f	volume fraction (–)
$\mathbf{f}$	thermal load (W)
g	standard acceleration of gravity (m/s^2)
H	domain height (m)
h	convective heat transfer coefficient (W/m^2 K)
$\mathbf{I}_2$	identity matrix of size 2 (–)
$\mathbf{K}$	thermal stiffness matrix (W/K)
k	thermal conductivity (W/m K)
L	heat sink length (m)
N_e	adjacent elements (–)
$\mathbf{N}$	shape function matrix (–)
Nu	Nusselt number (–)
$\mathbf{n}$	outward unit normal vector (–)
Pr	Prantl number (–)
p	penalization factor (–)
Q	total heat input (W)
q	boundary surface heat flux (W/m^2)
R_{th}	thermal resistance (K/W)
T_∞	ambient temperature (K)
T_b	base temperature (K)
$\mathbf{T}$	vector of temperature (K)
V_Ω	volume of computational domain (m^3)
W	heat sink width (m)
w_c	channel spacing of plate-fin (m)
$\mathbf{x}_i$	spatial location of element I (–)

Greek symbol

α	thermal diffusivity (m^2/s)
β	volumetric thermal expansion coefficient (1/K)
Γ	boundary of design domain (–)
γ	relative density (–)
η	fin efficiency (–)
μ	dynamic viscosity (N s/m^2)
v	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
Υ_Ω	volume of the computational domain (–)
Ω	computational domain (–)

Subscripts

conv	convection
eff	effective
f	fluid
fin	fin
ins	insulation
L	heat sink length
q	heat flux
S	solid
sur	surface

the corresponding local distribution of the heat transfer coefficient. Alexandersen et al. [11] extended their own 2-D conjugate heat transfer problem to 3-D and presented a density-based topology optimization method based on full-blown 3-D modelling. These methods are considered to be the best way to obtain shape-dependent heat transfer coefficients in terms of accuracy. However, the computational cost for the employment of a full-blown fluid model is rather high. Thus, it is necessary to develop a proper surrogate model that offers simple but fairly accurate predictions of the heat transfer coefficient even with a low computational cost.

In this study, optimum design of natural convection heat sinks is suggested by topology optimization in a 2-D computational domain. To account for shape-dependent variation of the heat transfer coefficient, a simple but accurate surrogate model that is generally applicable to natural convection problems is proposed. To validate the proposed surrogate model, topology optimization is performed with and without the surrogate model under a rectangular domain for which the optimum geometry for a plate-fin heat sink has been suggested. After the validation, the surrogate model is applied to a sample problem to obtain a new design for a heat sink. To improve the manufacturability of the conceptual design obtained by topology optimization, guidelines for design simplification are suggested. Finally, the thermal performance of the simplified heat sinks is compared to that of a conventional heat sink using a numerical simulation.

2. Topology optimization method

In this study, the topology optimization method is used with a newly suggested surrogate model accounting for the variation of the heat transfer coefficient in natural convection. In this section, the surrogate model is proposed based on the mathematical formulation for the conduction-convection problem. The optimization procedure including the calculation of the heat transfer coefficient distribution is then introduced. Finally, the surrogate model is validated by performing topology optimization on a rectangular domain for which the optimum geometry for a plate-fin heat sink has been suggested.

2.1. Finite element method for the conduction-convection problem

Fig. 1 shows the computational domain (Ω) and boundaries where equations for the finite element method are formulated.

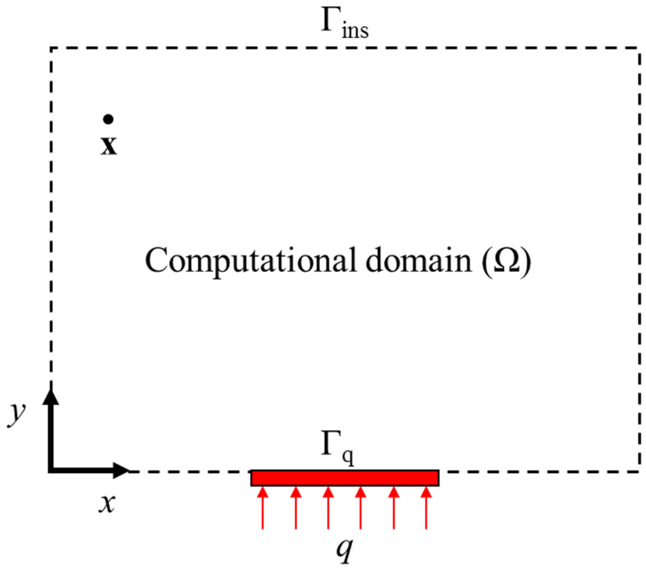

Fig. 1. 2-D computational domain for topology optimization.