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Double Diffusive Natural Convection Flow Over a Wavy Surface Situated in a Non-absorbing Medium

Sadia Siddiqa [a,b,*], M.N. Abrar [c], M.A. Hossain [d], Rama Subba Reddy Gorla [e]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Kamra Road, Attock, Pakistan
[b] Dipartimento di Ingegneria, Università degli Studi di Napoli "Parthenope", Centro Direzionale, Isola C4, Napoli, Italy
[c] Department of Mathematics, Capital University of Science and Technology, Zone V Express Way, Islamabad, Pakistan
[d] University of Dhaka, Dhaka 1000, Bangladesh
[e] Department of Mechanical & Civil Engineering, Purdue University Northwest, Westville, IN 46391 USA

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ABSTRACT

This analysis discuss the influence of heat and mass transfer on natural convection boundary layer flow of thermally radiating wavy surface. To make the surface radiating; Stephan-Boltzmann law is used in the boundary conditions. Therefore, convection and thermal radiation interact simultaneously with the aid of Stephan-Boltzmann law and ultimately producing highly nonlinear boundary conditions. The governing coupled boundary layer equations are switched into suitable form by using primitive variable formulations over which two-point iterative finite difference scheme is applied to obtain the unknown quantities. Physical quantities like wall shear stress, rate of heat transfer and rate of mass transfer are expressed graphically by varying significant emerging parameters: surface radiation (R), radiative length parameter (ξ), buoyancy ratio parameter (N) and amplitude of the wavy surface (α). Comparison of numerical results is also done in tabular form with the earlier study of Siddiqa et al. (2013) in order to validate the results.

1. Introduction

The natural convection along a heated uneven surface has attained a special interest due to its practical applications in high temperature aerodynamics [1]. It is reported well in literature that uneven/irregular surfaces transfer more heat as compared to smooth or flat surface. Due to this characteristic, rough surfaces are preferred in many industrial based applications, for instance, solar collectors, condensers in refrigerators, cavity wall insulating systems, grain storage containers, and industrial heat radiators are the few examples where irregular surfaces are used extensively [2]. Distribution of heat transfer along a semi-infinite vertical wavy surface of Newtonian fluid was initially discussed by Yao [3]. The effect of natural and mixed convection boundary layer flow over a wavy surface was separately studied by Moulic and Yao [4,5]. The effect of spatially stationary surface waves on the forced convection induced by a moving surface was examined by Rees and Pop [6]. They solved the problem numerically by using well known Keller-box method. Hossain and Rees [7] further exploit the idea of mass diffusion over the vertical wavy surface by considering natural convection flow. They also obtained the solutions numerically by using implicit finite difference method together with block tri-diagonal Keller-box scheme. The authors in [7] discussed the solutions for wide range of Schmidt number (0–1500) and concluded that the presence of surface waves serve to thicken the boundary layer and reduce the surface rate of heat transfer, concentration gradient and shear stress. Jang et al. [8] presented the numerical study of natural convection heat and mass transfer along a vertical wavy surface. Simple coordinate transformation was employed to transform the wavy surface to a flat plate and a marching finite-difference scheme was used in order to obtain the solutions. Later, numerical study of mixed convection heat and mass transfer along a vertical wavy surface was analyzed by Jang and Yan [9]. Further, forced convection confined between wavy channel was investigated by Mohamed et al. [10] in which they examined the effects of Prandtl number, Reynolds number and amplitude of the corrugation on transfers. Three-dimensional periodically developed incompressible flow in a triangular wavy fin-and-tube heat exchanger was numerically simulated by Cheng et al. [11]. They adopted a novel CLEARER algorithm and specifically noted the effects of the wavy angle, fin pitch, tube diameter and wavy density on pressure drop and heat transfer characteristics under different Reynolds number.

It is well known that generally thermal radiation play its role in the enhancement of surface heat transfer particularly in situations where convective heat transfer coefficients are small. A number of

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* Corresponding author at: Department of Mathematics, COMSATS Institute of Information Technology, Kamra Road, Attock, Pakistan.

E-mail address: saadiasiddiqa@gmail.com (S. Siddiqa).

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Nomenclature

C_f	coefficient of local skin friction
D	mass diffusivity (m^2/s)
g	acceleration due to gravity (m/s^2)
Gr_L	Grashof number
L	characteristic length of the vertical surface (m)
Nu	coefficient of local Nusselt number
Sh	coefficient of local Sherwood number
Pr	Prandtl number
q_w	uniform heat flux (W/m^2)
m	uniform mass flux ($kg/m^2\ s$)
R	surface radiation parameter
T	temperature of the fluid in the boundary layer (K)
T_w	temperature at the surface (K)
T_∞	temperature of the ambient fluid (K)
$\tilde{u}, \tilde{v}$	dimensional fluid velocities in the $\tilde{x}$- and $\tilde{y}$-direction, respectively (m/s)
u, v	dimensionless fluid velocities in the x- and y-direction, respectively
$\tilde{x}, \tilde{y}$	dimensional Cartesian coordinates (m)
x, y	dimensionless Cartesian coordinates

Greek letters

$\bar{\alpha}$	thermal diffusivity (m^2/s)
$\tilde{\alpha}$	dimensional amplitude of the wavy surface (m)
α	dimensionless amplitude of the wavy surface
β	volumetric coefficient of thermal expansion (1/K)
ϵ	emissivity
κ	thermal conductivity (W/mK)
μ	dynamic viscosity (kg/ms)
ν	kinematic viscosity (m^2/s)
ρ	density of the fluid (kg/m^3)
σ_e	Stephan-Boltzmann constant ($W/m^2\ K^4$)
θ	dimensionless temperature function
τ_w	skin friction
ξ	radiative length parameter

Subscripts

w	wall condition
∞	ambient condition

mathematical models are used in literature which either introduces thermal radiation heat flux in the energy equation or sometimes in the boundary conditions. Some models even bring in separate governing equations for thermal radiation. Numerous authors, for example [7,12–19], investigated thermal radiation effects by keeping in view Rosseland diffusion approximation. It is worthy to mention that diffusion approximation is particularly valid in the interior of the medium but not near the boundaries. Therefore, heat transfer processes involved in [7,12–19] are accompanied by the absorption, emission, and scattering of radiation by the fluid. The interaction between radiation and convection can also be originated by assigning a boundary condition on a heat radiating surface over which the fluid flows. In this view, Salomatov and Puzyrev [20] took variable thermo-physical properties and examined the behavior of non-absorbing fluid under natural and forced convection conditions. Later on, Martynenko et al. [21] investigated the influence of heat emitting vertical surface on the free convection flow in a non-absorbing medium. The effect of radiation on free convective heat liberation from the surface of a vertical cylinder located in a transparent medium was studied by Sokovishin and Shapiro [22]. They showed that the radiative component of thermal flux equalizes the surface temperature. More recent analysis of Siddiqa and Hossain [23] and Siddiqa et al. [24] obtained numerical solutions for liquid metals by taking the constraints of non-absorbing medium into account.

However, in nature and in many engineering applications there are many transport processes which are governed by the joint action of both thermal and mass diffusion. The engineering applications include chemical reactions in a reactor chamber, chemical vapor depositions of solid layers, combustion of atomized liquid fuels and dehydration operations in chemical and foundry plants (see Gebhart and Pera [25,26]). Detailed information on simultaneous heat and mass transfer in convection boundary layer flows over plates can be found in Refs. [27–31].

In the present analysis, heat and mass transfer effects are observed for the non-absorbing medium over a semi-infinite vertical wavy surface. Previously, Siddiqa et al. [24] discussed the case of heat transfer only. Therefore, motivation behind this paper is to extend the work of [24] by considering mass transfer as well. Coordinate transformations known as primitive variable formulations (PVF) are employed to transform the boundary layer equations into parabolic partial differential equations. System of equations obtained from PVF is simulated numerically via implicit finite difference method that uses Thomas algorithm as a solver. These numerical results are obtained for liquid metals and particularly lead (Pb) is used for which Prandtl number Pr is 0.024 operating at 644 K. The physical properties of liquid metals give them unique advantages in many technological applications. Specially high heat capacities, excellent thermal conductivities and generally low viscosities make them ideal as a heat transfer media. Choice of taking lead as a working fluid is due to the fact that it can permit reduction of costs for operation maintenance. More interestingly, lead-based coolants have significantly higher boiling points as compared to other liquid metals, meaning a reactor can be operated without risk of coolant boiling at much higher temperature, which improves thermal efficiency. Numerical results are obtained in terms of shear stress, rate of heat transfer and rate of mass transfer for important parameters that occur while addressing the issue.

2. Mathematical formulation

In this article, consideration has been given to steady, two dimensional, natural convection boundary layer flow of a viscous, incompressible fluid along a semi-infinite, thermally radiating wavy surface. The conjugate effects of thermal and mass diffusion are discussed here. The fluid flow is driven by both thermal and solutal buoyancy forces. The fluid is placed in the region of optically transparent medium and the process of radiation absorption, emission and scattering is ignored. The heat transfer for non-absorbing medium is determined independently for the radiative and convective components. For the given value of wall heat flux such independence of the two heat transfer modes does not hold. The heat flux from the wall is carried by the radiative component, which depends on the local temperature, as well as by the convective and conductive components. The latter determine the wall temperature and therefore affect the radiation. All the fluid properties are considered constant except density, which obeys Boussinesq approximation and thus the density difference is indispensable to the natural convection motion and must be retained where they appears in the body force term (i.e., term multiplied by g, the acceleration due to gravity), but elsewhere the density variation is considered to be small enough and is neglected. Our detailed numerical work will assume that the surface exhibits