International Journal of Heat and Mass Transfer 109 (2017) 215–222

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A scaling analysis for electrohydrodynamic convection with variable thermophysical and electrical properties

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ARTICLE INFO

Article history:
Received 26 July 2016
Received in revised form 15 December 2016
Accepted 27 January 2017

Keywords:
Electrohydrodynamic
Convective transport
Nusselt number
Rayleigh number

ABSTRACT

Electrohydrodynamic (EHD) effects cause fluid motion when an external electric field is imposed on a dielectric fluid medium. In this work, a detailed scaling analysis of EHD convective transport is presented, taking into account the interplay of electrophoretic, dielectrophoretic, and electrostriction forces and the variable nature of the concerned thermophysical and electrical properties. Our results bring out the scaling relationships for the Nusselt number as a function of the relevant electrical Rayleigh number (Ra_{EL1} and Ra_{EL2}), for different operating regimes, and provide important insights into the relative dominances of the different forces. The Nusselt number is found to scale with $Ra_{EL1}^{0.25}$ when the Coulomb force dominates, and with $Ra_{EL2}^{0.333}$ when electrostriction force dominates. Numerical simulations are also conducted for some representative test cases and the results are found to corroborate the scaling theory.

1. Introduction

Natural convection heat transfer is regularly encountered in various engineering and industrial applications [1,2]. Being solely driven by buoyancy induced flows and not requiring any prime movers, this mode of heat transfer is often the preferred and reliable mechanism of thermal transport. However, there are certain circumstances where natural convection becomes weak due to reduced gravity, such as in outer space applications. Under such circumstances, electrohydrodynamically (EHD) induced convection can be considered as a feasible method for augmenting thermal transport. In addition, the thermal performance can be significantly enhanced by the application of nanofluid in presence of EHD field [3–7]. On the other hand, the combined effect of buoyancy and electrohydrodynamic driven convection has other applications, such as solar energy heater, thermal management of main frame computer, and transformer motor [8]. Electrohydrodynamic (EHD) field is induced due to the polarization of a dielectric fluid. Coupling of convective heat transfer with EHD force is an effective method of enhancing heat transfer because of no moving parts, low power consumption and noise, and rapid and smart control [9] . This interdisciplinary field deals with electrostatics, fluid flow, and thermal fields [10].

Yabe et al. [11] studied EHD enhanced natural convection between wire and plate electrode and showed good agreement between numerical and experimental data. Velkoff and Godfrey [12] investigated convective heat transfer between a plate and an electrode, and found a large increase in heat transfer at low stream velocities; however, the same enhancement was absent at high velocities. Several studies have been reported on EHD enhanced natural convection inside an enclosure [13–15]. EHD enhanced oil flow in a heated annulus was investigated by Fernandez and Poulter [16]. It was reported that radial motion of the oil is strongly influenced by EHD force. A correlation was also reported between the electric current and the EHD enhanced heat transfer coefficient. Kasayapan and Kiatsiriroat [17,18] reported their numerical results on optimized electrode arrangement inside a horizontal channel using computational fluid dynamics, and showed the best heat transfer enhancement on optimization of electrode arrangement. Owsenek and Seyed-Yagoohi [19] studied EHD enhanced natural convection experimentally, over a heated flat plate using a needle-plate assembly. It was found that the heat transfer rate increased more than 25 times when compared to pure natural convection. Optimization was made based on applied voltage and electrode height. The efficiency of the needle-plate assembly was improved with increasing height, and finally became independent above a critical value of 5-cm.

Many studies were conducted to characterize the interplay of electric force with the buoyancy force induced from density differences. Kronig and Schwarz [20] introduced a characteristic electrical number (EL), which can be interpreted as the ratio of electric to viscous forces and is analogous to Grashof number in natural convection. Therefore, the corresponding electric Rayleigh number can be expressed as $Ra_{EL} = EL \times$ Pr. Ahsmann and Kronig [21] modified this number for liquid as $EL = (\rho \varepsilon_0 \frac{d\kappa}{dT} E^2 H^2)/\mu^2$. Rutkowski [22]

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.01.104
0017-9310/

correlated increased Nusselt number with electric Rayleigh number as $\Delta Nu = 0.68 Ra_{EL}^{0.4}$ for $1.0 \times 10^4 \leqslant Ra_{EL} \leqslant 3.0 \times 10^5$. Pascual et al. [23] investigated EHD natural convection from heated platinum wires immersed in a bath of R-123. From their measurements, they correlated the increment in Nusselt number (over the same for classical buoyancy driven convection) with electrical Rayleigh number as $\Delta Nu = 0.089 Ra_{EL}^{0.365}$ for $4 \times 10^3 \leqslant Ra_{EL} \leqslant 8 \times 10^8$. Recently, Marucho and Campo [24] addressed a theoretical derivation on EHD enhanced natural convection under a uniform electric field. They concluded that the total Nusselt number cannot be expressed as an algebraic sum of the individual *Nu* due to EHD and natural convection. Nahavandi and Mehrabani-zeinabad [8] examined EHD effect on natural convection through a vertical channel with water as a liquid medium. It was found that with applied potential of 100V and temperature difference of 40 K, the heat transfer rate was increased significantly. Importance of electrostriction over Coulomb forces was also presented. However, relative dominance of various forces of electric field is not well established so far.

When an electric field is applied on a dielectric liquid having permittivity variation with temperature, electrostrictive forces are likely to become important, compared to Coulomb force acting on the free charge. Implications of these electrical forces, however, are by no means obvious. This is because, depending on the polarity of the free charge and the direction of the gradient of temperature and square of the electric field strength, the interplay between the viscous and the electrical forces may alter non-trivially. This, in turn, may alter the heat transfer characteristics in a non-intuitive manner, which is yet to be brought out in the literature. In the present study, we attempt to outline a scaling analysis, in an effort to show how these forces interplay with each other, with regard to the consequent Nusselt number characteristics. We also corroborate our scaling estimates with comprehensive numerical simulations.

2. Problem formulation

2.1. System description

In the present work, numerical simulations have been conducted to study EHD driven convection in a side heated/cooled rectangular cavity (Fig. 1). One third of the left wall at the middle is heated isothermally. The cavity is filled with dielectric fluid (water). A half circular cut surface is located just opposite of the heated wall. The surface of the half circular region is isothermally cooled. All the other walls (except heated part of the left wall and cooled circular surface of right wall) of the cavity are insulated. DC voltage is applied through the wire electrode located at the centre of the cavity. The left wall of the cavity is electrically grounded, whereas the remaining walls are electrically insulated. The cavity has a large depth along the third direction, so that a 2-D model can be employed. The coolant in the cavity dissipates the generated heat. Externally applied EHD field forces the coolant to move over the heated surface and create a recirculation between the heated and cooled wall.

The coordinate system, along with various dimension of the cavity, is shown in Fig. 1. Here *H* is the length of the heated portion of the left wall and forms an important length scale for characterizing the physics of the problem under investigation. Hence, it is taken as reference length. All the other dimensions are normalized with respect to this length.

2.2. Governing equations and boundary conditions

The electric force density in presence of an electric field on a dielectric fluid is given by:

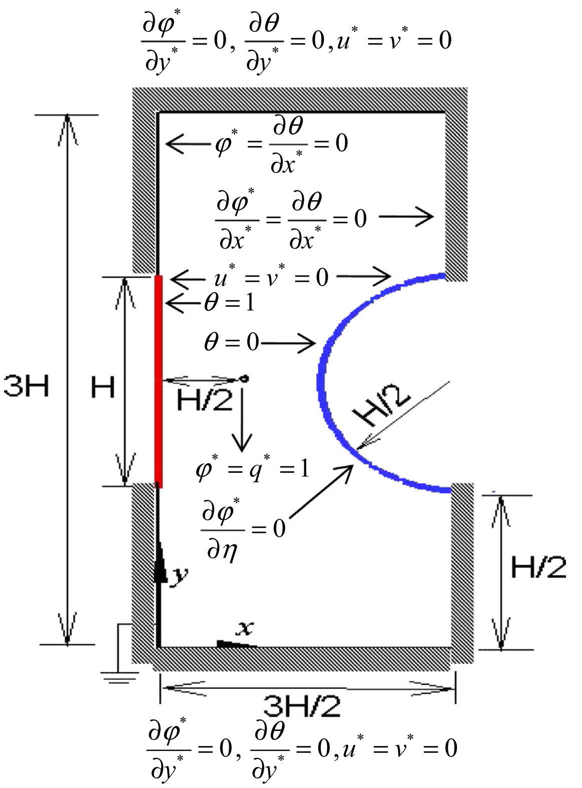

Fig. 1. Schematic representation of the system.

$$\mathbf{F}_v = q\mathbf{E} - \frac{1}{2}\mathrm{E}^2 \nabla \varepsilon + \frac{1}{2}\nabla\left[\rho \mathbf{E}^2 \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T\right] \tag{1}$$

where **E** is the electric field, ε is the permittivity of the medium, and ρ is the density of the medium. The electrophoretic term (1st term of the right hand side of Eq. (1) represents the Coulomb force imposed on free charge density *q*. The 2nd term is the dielectrophoretic force density, which depends on local gradient of fluid permittivity. The 3rd term is the electrostrictive force component related with non-uniform electric field.

Influence of EHD on heat transfer and fluid involves simultaneous coupling of free charge, electric field, fluid flow and heat transfer. To begin with, electric field equations must be solved first. The combination of Maxwell's relation and Gauss's law yields

$$\nabla \cdot (\varepsilon \mathbf{E}) = q \tag{2}$$

where ε and *q* are permittivity and electric charge density, respectively. Again the potential (φ) is related with **E** as:

$$\mathbf{E} = -\nabla \varphi \tag{3}$$

From charge conservation equation

$$\nabla \cdot \mathbf{J} + \frac{\partial q}{\partial t} = 0 \tag{4}$$

where **J** is current density. It is expressed as

$$\mathbf{J} = q\mathbf{V} + qb\mathbf{E} - D\nabla q \tag{5}$$

Here, $q\mathbf{V}, qb\mathbf{E}$ and $D\nabla q$ are the charge convection, electric conduction, and diffusion term, respectively. *b* and *D* (order of 10^{-14} [25]) are the ion mobility and diffusion coefficient, respectively. Typical order of b is 10^{-7} [26]. To couple the electric field with the flow field and heat transfer, electric body force and Joule heat are included in the momentum and energy equations, respectively. For non polar dielectrics, using Clausius-Mosotti law [27]