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Dynamics of unsteady heat transfer in pulsating flow across a cylinder

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ABSTRACT

The unsteady heat transfer between a cylinder and pulsating cross-flow is investigated for small perturbations of flow velocity. In this regime the cycle-averaged heat transfer is constant and fluctuations of flow variables can be described as linear, time-invariant dynamics. Numerical simulation of the response to a sudden increase of the free stream velocity allows to visualize and interpret physically the flow and heat transfer dynamics. Broadband excitation combined with linear system identification yields quantitative predictions of the frequency response of heat transfer over a range of Reynolds and Strouhal numbers. It is concluded that the heat transfer dynamics are governed by several time scales, corresponding to the response times of the velocity field and temperature field, respectively. The interaction of the different time lags leads to a non-trivial dependence of the heat transfer frequency response on Strouhal and Reynolds numbers. The frequency response functions exhibit a low-pass behavior with vanishing amplitudes and a phase lag slightly above $-\pi/2$ at high Strouhal numbers. Excess gain above the quasi-steady-state value of the heat transfer frequency response is observed for Strouhal numbers of order unity and Reynolds numbers of order ten.

1. Introduction

The heat transfer of a cylinder in cross-flow is one of the archetypal problems in thermo-fluid mechanics. Due to its fundamental importance and technical relevance, different aspects of this configuration have been studied intensively in a large number of publications, review articles and textbooks, see e.g. [1–5] and references therein.

The majority of studies addresses steady, laminar flow; or flows at larger Reynolds numbers, where intrinsic unsteadiness arises from flow instability or turbulence. Configurations where flow unsteadiness results from impulsive or pulsating modulation of the free stream have not been investigated as intensively. Nevertheless, a number of pertinent studies have been carried out focusing either on the average heat transfer (heat transfer enhancement) [6–8] or the dynamic behavior [9–14]. They are of fundamental interest for thermo-fluid-dynamics and of relevance for applications like hot-wire anemometry [15] or the Rijke tube [16,17]. In the latter case, which is of particular interest for the authors of the present paper, a heated wire mesh in pulsating cross-flow can drive a self-excited thermo-acoustic instability. Stability limits and pulsation amplitudes depend in a sensitive manner on the dynamic response of heat transfer to velocity fluctuations.

In a landmark paper, Lighthill [9] studied the response of laminar skin friction and heat transfer to fluctuations in the stream velocity. Lighthill presented approximate solutions for near-wall velocity and temperature profiles for the case of small pulsation amplitudes (linear regime) and evaluated the corresponding displacement thickness, skin friction and heat flow rate, respectively. For the case of a cylinder in low Reynolds number cross-flow, the heat transfer frequency response in terms of amplitude reduction and phase lag was determined. Lighthill states that his solution "applies only at Reynolds numbers for which the boundary-layer approximation has some validity (say $R > 10$)" and "in the range of Reynolds number for which a laminar boundary layer exists" [9]. This study was thereafter extended to compressible flow by Gribben [18].

Lighthill already suggested that the unsteady response of the heat transfer rate to the perturbation of free stream velocity is determined by the adaptation time of the viscous and thermal boundary layers. For harmonic perturbation, this time lag controls the phase of the heat transfer frequency response function. For low frequencies, the time lag was estimated as one fifth of the ratio of cylinder diameter and free stream velocity. However, since it was unavoidable to introduce severe approximations to solve the equations of motion, Lighthill himself cautioned that his results would only yield solutions of limited accuracy and range of validity and

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Nomenclature

Roman letters

$\dot{q}$	heat flux density
A	area
a	speed of sound
B	transfer function polynomial
b	coefficient
c_τ	time constant
c_p	isobaric specific heat capacity
d	cylinder diameter
e	stochastic disturbance
F	transfer function polynomial/Force
f	coefficient
G	transfer function/frequency response
g	impulse response
h	step response
i	imaginary unit
K	steady state gain
k	time increment
L	length
l	counter
N	total number of datapoints/number of cells
n	interaction index or order of polynomial
P	modified pressure
p	pressure
q	time shift operator
s	parameter (frequency)
T	temperature
t	time
T_s	sampling time
u	velocity in x-direction
V	volume
v	velocity in y-direction
x	noise free model output
x, y, z	coordinates
y	model output
$\vec{n}$	surface normal vector

Greek letters

α	heat transfer coefficient
χ	thermal diffusivity
δ	perturbation parameter/boundary layer thickness/difference to steady state
$\dot{\omega}$	heat source density
ϵ	small parameter/amplitude
λ	thermal conductivity
ν	kinematic viscosity
ω	angular frequency
Φ	NRMSE-fit
ϕ	polar angle/cell face flux
ρ	density
σ	growth rate
τ	dimensionless time
θ	parameter vector
ε	prediction error

Dimensionless groups

Bi	Biot number
CFL	CFL number
Ec	Eckert number
Fo	Fourier number
He	Helmholtz number
Ma	Mach number
Nu	Nußelt number
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
Sr	Strouhal number
Wo	Womersley number

Superscripts

$\hat{}$	estimated
$\bar{}$	temporal average
$'$	fluctuating

Subscripts

0	steady-state
∞	ambient
c	cylinder
d	domain
f	fluid
m	mean
s	solid/step
w	surface (wall)

applicability. Lighthill's results were only validated for velocity amplitudes in the unsteady boundary layer [19], but never with respect to heat transfer. Presumably, this is due to the difficulties of time-resolved measurements of heat flow rates. Nevertheless, Lighthill's estimate for the time lag has since been used in many studies of thermoacoustic instability of the Rijke tube [20–28]. In particular, Subramanian et al. [25] carried out a comprehensive bifurcation analysis of thermoacoustic instability. In this context, Mariappan[1] reports that with a time lag up to three times as large as Lighthill's value, much better agreement of stability analysis with experimentally observed stability limits is obtained.

Several authors set out to improve Lighthill's analysis. Gersten [29] revisited Lighthill's analysis and re-cast it into the Falkner-Skan equation, introducing first and second order perturbations. For two cases, i.e. the stagnation flow (Hiemenz layer) and the flow over a flat plate (Blasius layer), Gersten develops transfer functions as power series of the Strouhal number Sr. Similar to Lighthill's approach the solution comprises separate approximations for low and high frequencies, respectively. A solution for compressible flow was considered by Gribben [30]. Telionis [31] summarizes advances in describing unsteady viscous flows, a whole chapter of his book is devoted to fluctuations imposed on a steady flow. Both analytical and numerical investigations applying the boundary layer equations are discussed. Most notable in the context of this work is the numerical solution of the unsteady boundary equations by Telionis and Romaniuk [32]. Although no transfer function was derived, relations between fluctuating and steady temperature gradients shed some light on the heat transfer behavior. In contrast to Lighthill's solutions, a peak in fluctuation amplitude at low Strouhal numbers was reported. Applying a series expansion to Oseen's solution for a cylinder in cross-flow at low Péclet numbers, Bayly [12] derives an expression for the unsteady heat transfer in this regime. This formulation yields a transfer function for creeping flow ($\mathrm{Re} < 1$ assuming $\mathrm{Pr} \approx 0.7$) around a cylinder.

The work of Kwon and Lee [13] is of particular relevance for the present study. A stream function/vorticity formulation was used to model incompressible, two-dimensional flow around a heater wire.

[1] Private communication, 2012.