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Grey-box model for pipe temperature based on linear regression

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ABSTRACT

Developing mathematical models describing pipe (or duct) temperature is of great importance, since pipes are unavoidable elements in most (hydraulic) heating systems, in which some heat transfer fluid flows/circulates between neighbouring working components (such systems are district, central or solar heating systems, etc.).

In the present study, the Newton's law of cooling is completed with a recent explicit equation determining the time delay of pipes. Based on measured data, the gained mathematical model, called physically-based model, describes the outlet (fluid) temperature of pipes with a convenient accuracy with respect to the practice.

A further model, called LR model, is worked out based on multiple linear regression. Based on measured data, the LR model can model the outlet temperature of pipes generally more precisely than the physically-based model if the flow rate is nonzero. In addition, the LR model has lower computational demand.

Since the physically-based model is still more precise under certain conditions, a third model, called grey-box model, is proposed as a combination of the physically-based and the LR model calculating every time according to the more advantageous one of them. Based on measured data, the grey-box model is the most precise model. In addition, this model has lower computational demand than the physically-based model.

1. Introduction

Developing mathematical models describing pipe (or duct) temperature is of great importance, since pipes are unavoidable elements in most (hydraulic) heating systems, in which some heat transfer fluid flows/circulates between neighbouring working components. District heating systems, central heating systems and solar heating systems can be mentioned here as particular examples.

In many works, the heat loss effect, and even more often, the time delaying effect, of pipes is neglected. Nevertheless, one or both of the above pipe effects must be often considered, for example, if the pipe is not so well insulated and/or the pipe is relatively long. This is the case for district heating systems [1,2] or e.g. in [3], where 5–10% more solar energy can be gained at a real solar heating system with differential control if the pipes are taken into account precisely enough.

As for most other working components, differential equations (DEs) are the most frequent mathematical models to model pipes. See, for example, [4], when an inverse method is used to estimate the inlet temperature on the basis of the governing DEs. In the simplest case, a pipe is modelled with a single ordinary differential equation (ODE) assuming homogeneous pipe (fluid) temperature (with respect to space) as the single state variable. The heat loss can be considered in such models [5], so they can be accurate enough on an average in long terms, but, if the transients are important, more sophisticated models should be used.

The most often model, describing both delay and heat loss, is the linear one dimensional partial differential equation (PDE) corresponding to heat transfer and plug-flow (that is, the mixing and temperature homogenization effects inside the pipe are neglected). In other words, this model is the one dimensional linear transport equation [6] describing the pipe (fluid) temperature supported with a member with respect to heat loss, optionally. This PDE is applied to model temperature distribution in solar collectors in [7,8], in heat exchangers with connecting pipes in case of neglected heat losses in [9], in chemical tubular reactor in [10] and in a district heating system in [2]. In the latter difference, the PDE is used to determine time delay directly as well. In [11], the PDE describing the temperature distribution inside the pipe is applied to a water heating equipment of pilot-scale in case of perfectly insulated pipes. By means of a simplifying procedure, the PDE is transformed

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Nomenclature

t	time, s;
x	space coordinate along the pipe, m
Time-dependent functions	
T	pipe temperature, °C;
T_a	ambient temperature of the pipe, °C;
T_{in}	inlet pipe temperature, °C;
T_{out}	outlet pipe temperature, °C;
$T_{out,meas}$	measured outlet pipe temperature, °C;
$T_{out,\mathrm{mod}}$	modelled outlet pipe temperature, °C;
v	(pump) flow rate in the pipe, $m^3\,s^{-1}$
Constant parameters	
A	area of pipe cross section, m^2;
c	specific heat capacity of the pipe fluid, $Jkg^{-1}\,K^{-1}$;
k	heat loss coefficient of the pipe, $Wm^{-1}\,K^{-1}$;
L	length of pipe, m;
V	pipe volume, m^3;
Δt	time period between successive measurements on the pipe, s;
ρ	mass density of the pipe fluid, kgm^{-3}

into two models for control purposes. One of them contains ODEs for perfectly mixed sections, while the other one is a length integrated model, determining the average of the temperature along the pipe. Similarly, PDEs describing temperature distribution are used for pipes and parallel-plate channels in [12] and for a basic natural circulation loop in [13]. If the one dimensional linear heat transfer equation corresponding to pipe temperature is applied to the moving "fluid element" inside the pipe, essentially, the well-known Newton's law of cooling [14] is gained.

Although, PDEs are usually more difficult to handle than ODEs and they cannot be solved exactly, solutions with desired precision can be generally gained by means of discretization methods. For example, in the TRNSYS software [15], which is widely used to simulate transient thermal processes in different heating systems, a pipe is divided into segments, each of which has homogeneous temperature and is modelled with an ODE. The pipes of district heating systems are discretized in [1], after which, the temperature is calculated numerically.

Although, PDEs can be generally solved numerically with desired precision, there is a problem of principle with respect to the (one dimensional) linear transport equation corresponding to pipe (fluid) temperature. Namely, if the (pump) flow rate is intermittent, that is, the flow rate is sometimes zero, the transport equation may have no classical solution or may have not unique solutions. Neither case satisfies the natural physical expectation on definiteness (see Remark 2.1 in [16] for more details). This problem is avoided with nonzero flow rate (e.g. in [7,17]) or with numerical solution (e.g. in [10]).

The problem of discontinuity can be overcome if heating systems affected with the delaying effect of pipes are modelled with delay differential equations (DDEs) [18,19]. Most references on thermal engineering problems deal with constant time delay. See, for example, [20,21] on DDEs with respect to heat conduction if the heat flux vector is endowed with time delay. In [22], the pipe outlet temperature is described in time by means of a delayed equation derived from the heat transfer PDE in case of constant flow rate. An explicit formula is given to express the time delay as a function of time in case of variable flow rate in [17], although, it can be used only if the flow rate is nonzero. The concept is improved in [16], where an explicit delay equation is proposed, which can be applied to intermittent flow rates as well.

So far, white-box models were discussed as they are founded on known physical phenomena. In case of black-box models, some experienced/measured correlations are represented empirically. In the present work, multiple linear regression (MLR) is used in constructing a black-box model to determine the outlet temperature of pipes as a function of the inlet and ambient temperatures. See [23], where solar collectors, as other working components of heating systems, are modelled by means of MLR.

The contributions are the following in details in the present study.

1. In the determination of the outlet (fluid) temperature, the Newton's law of cooling is used to model the heat loss to the ambiance of the pipe completed with the explicit delay equation of [16] to determine the time delay. This white-box model is called physically-based model henceforth.
2. Furthermore, an MLR based model, called LR model, is worked out to determine the outlet temperature of pipes as a simple linear function of the inlet and ambient temperatures. It is presented based on measured data that the LR model is generally more precise than the physically-based one if the flow rate is nonzero. In addition, the LR model has lower computational demand.
3. Since the physically-based model is still more precise under certain conditions, a third model, called grey-box model, is proposed as a combination of the physically-based and the LR model calculating every time according to the more advantageous one of them. Based on measured data, the grey-box model is more precise than any of the other two models. In addition, the grey-box model has lower computational demand than the physically-based model.

The Matlab software [24] has been applied in this work to carry out the needed calculations.

The organization of the paper is the following: Section 2 gives common features on the pipe operation, the measurements and the modelling corresponding to all of the studied models. In Section 3 and 4, the physically-based and the LR model are constructed and their identification and validation are given based on measured data. The grey-box model is proposed and validated in Section 5. The detailed comparison of the models is given in Section 6. Conclusions and recommendations for future research works are given in Section 7.

2. Common features on pipe operation, measurements and modelling

2.1. Pipe operation

Fig. 1 shows the pipe to be modelled.

Obviously, it takes a certain time for the "fluid element" leaving the pipe inlet at time τ to reach the outlet at time t. That is, the delay d equals to $t-\tau$. If the (pump) flow rate in the pipe v is not constant but a function of time $v(t)$, then $\tau(t)$ also depends on time t, as well as the delay $d(t)$, see Eq. (1).

$$d(t) = t - \tau \qquad (1)$$