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Conjugated heat transfer from a uniformly heated plate and a plate fin with uniform base heat flux

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ABSTRACT

The relationship between maximum temperature and heating rate is determined for an internally heated flat plate, which is cooled on both sides by either forced or natural convection. Convection heat transfer is calculated using the energy integral boundary layer equations instead of solving governing partial differential equations numerically. The results are presented in simple analytical correlations.

Solutions to a heated plate were used in the study of a plate fin with a uniform heat flux at the base in cases of forced and natural convection. New simple analytical equations for the maximum fin temperature were derived using the result on the heated plate. Solutions to a fin with uniform base heat flux include also those to a uniform fin base temperature as limiting cases. The validity of the simple new solutions to an internally heated plate and a fin with a uniform heat flux at the base were verified by comparing results to numerically obtained conjugated heat transfer solutions, analytical solutions, and to some experimental data for a heated plate.

1. Introduction

Finned heat sinks are commonly used to enhance heat transfer in many applications where waste heat is a problem. Heat sinks must be optimized, for example, to extend the lifetime of electronic components in conditions where heat flux densities are constantly growing. The miniaturization of devices sets further requirements for compact and lightweight heat sink design.

In convectively cooled systems, a thermally optimal isothermal condition can be achieved for heat sources by placing them more densely at upstream locations [1,2] or by enhancing heat spread, for example, with a vapor chamber inside a heat sink [3]. However, when heat sources are flush-mounted, for example, due to space limitations, those at downstream locations are prone to overheating. In this case, fin base has uniform heat flux rather than uniform temperature. The performance and optimum geometry of single cooling fins are well known for fins attached on an isothermal surface [4–6], but the uniform heat flux condition has not been studied to an equal extent. Culham et al. [7] present some results for a plate fin with a uniform base heat flux for natural convection but comprehensive solutions to both forced and natural convection do not exist.

The coupling of convection and conduction, which governs fin performance, has been studied extensively for simple flat plate geometries [8]. In a classical academic problem studied by Luikov [9], a flat plate was heated at uniform temperature on one side while the other side was cooled by forced or natural convection. This problem was recently solved [10]. Another similar basic problem is the plate heated with uniform heat generation, which represents a board with flush-mounted heat sources. The literature contains various solutions to this problem for forced [11–14] and natural [15–17] convection cooling, yet most of them are difficult to apply in applications.

In this paper, we present two apparently separate problems, which, in fact, are connected. In Section 2, we study the fundamental problem of an internally heated plate cooled by either forced or natural convection. Our approach is to calculate numerically the plate temperatures and give simple correlations for the heat transfer coefficients. In Section 3, we demonstrate how a solution to a heated plate can be used in a fin model to take into account a uniform heat flux boundary condition at the fin base. This is the first complete treatment of convection cooled plate fins with a uniform heat flux boundary condition at the base.

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Nomenclature

c_p	fluid specific heat at constant pressure, J/kg K
g	gravitational acceleration, m/s^2
h	constant heat transfer coefficient, W/m^{2}K
h_{eff}	effective heat transfer coefficient of heated plate
$h_{L,H}$	local heat transfer of isoflux surface, Eq. (2.5)
$h_{m,T}$	mean heat transfer coefficient of isothermal surface, Eq. (2.7)
k_f	fluid thermal conductivity, W/m K
k	solid thermal conductivity, W/m K
l	fin height, m
L	plate or fin length in flow direction, m
Pr	Prandtl number, $\rho\nu c_p/k_f$
$q(x)$	local surface heat flux, Eqs. (2.2) and (2.3), W/m^2
q_b	uniform base heat flux, W/m^2
q_{ref}	reference heat flux for natural convection, Eqs. (2.6) and (3.6), W/m^2
q_y	conduction heat flux in y -direction, W/m^2
s	Dummy integration variable in Eqs. (2.2) and (2.3), m
t	plate or fin thickness, m
T	plate or fin temperature, K
T_∞	ambient temperature, K
U_∞	ambient flow velocity, m/s
x	coordinate in flow direction, m
X_*	conduction–convection parameter for plate, Eq. (2.4)
X	conduction–convection parameter for fin, Eq. (3.11)
y	coordinate normal to base, m

Greek symbols

β_T	fluid thermal expansion coefficient, K^{-1}
θ	temperature excess, $T - T_\infty$, K
θ_{max}	maximum temperature excess, K
ν	kinematic viscosity, m^2/s
ρ	fluid density, kg/m^3
ϕ'''	volumetric heat generation rate, W/m^3
Φ	total heat transfer rate of fin, q_bLt, W

2. Heat transfer from a uniformly heated plate

We begin by studying heat transfer from a uniformly internally heated thin plate, which is located in a free stream, as in Fig. 1a, or in a quiescent fluid, as in Fig. 1b. We seek to determine the maximum plate temperature as a function of heating rate, plate geometry, and type of convection. The maximum temperature, which is usually subject to constraints in applications, is found at the downstream end due to the growth of boundary layers in the flow direction.

2.1. Governing equations

In a thin plate with high thermal conductivity, temperature variations across the plate thickness can be ignored, and the plate temperature is governed by the balance of heat generation, convection, and one-dimensional conduction along the plate:

$$kt\frac{\partial^2\theta(x)}{\partial x^2} + t\phi''' = 2q(x), \tag{2.1}$$

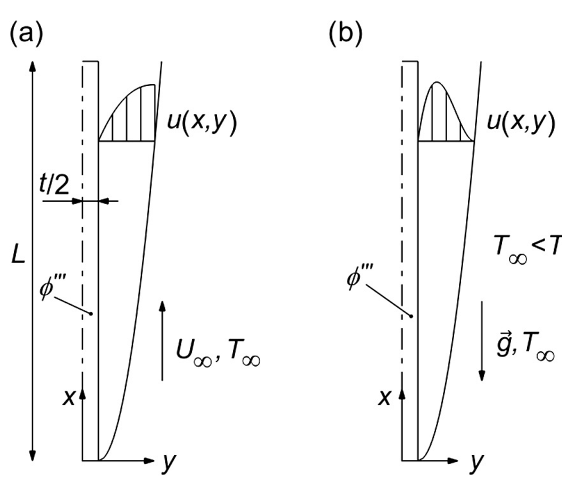

Fig. 1. One half of internally heated thin plates cooled by forced (a) and natural (b) convection.

where $\theta(x) = T(x) - T_\infty$ is the temperature excess, ϕ''' the heat generation, and $q(x)$ the convective heat flux. The leading and trailing edges are assumed to be insulated; this assumption agrees with experimental data [12].

For *forced convection*, the surface heat flux $q(x)$ in Eq. (2.1) can be obtained by using the energy integral equation ($\Pr \gtrsim 1$) [18]

$$q(x) = C\frac{k_f}{x}\left(\frac{U_\infty x}{\nu}\right)^m \Pr^n\left(\theta(0) + \int_0^x\left[1-\left(\frac{s}{x}\right)^\gamma\right]^{-\beta}\frac{\partial\theta(s)}{\partial s}ds\right), \tag{2.2}$$

which takes into account the variation in surface temperature in heat transfer. The values of the constants in Eq. (2.2) are given in Table 1 for laminar and turbulent boundary layers. The surface heat flux due to *natural convection* on a vertical plate is coupled also with the magnitude of the difference in temperature, because temperature differences drive the flow. The first model to couple surface temperature variation and convective heat flux was developed by Raithby and Hollands [19], who used an analogy between natural convection and condensation. We use a somewhat more accurate model by Lee and Yovanovich [20]:

$$q(x) = 0.353\frac{k_f}{x^{1/2}}\left(\frac{g\beta_T}{\nu^2}\int_0^x\theta(s)ds\right)^{1/4} \times\left(\theta(0) + \int_0^x\left[1-\left(\frac{s}{x}\right)^{9/8}\right]^{-11/24}\frac{\partial\theta(s)}{\partial s}ds\right). \tag{2.3}$$

Eq. (2.3) is valid for an isothermal plate with a laminar boundary layer of gas ($\Pr = 0.7$) when $\mathrm{Ra} = g\beta_T\theta L^3\nu^{-2}\Pr \lesssim 10^9$. In turbulent boundary layers, where $\mathrm{Ra} \gtrsim 10^9$, variation in surface temperature affects the heat transfer coefficient very little, and the equations of the heat transfer coefficients of uniform heat flux and temperature are similar [18]. In addition, in electrical applications cooled by natural convection, turbulent boundary layers seldom occur.

2.2. Limits of the heat transfer coefficient

On the surface of a heated plate, the distribution of the heat transfer coefficient depends on the ratio of the thermal resistance of conduction along the plate (L/kt) and that of the boundary layer ($1/Lh_{L,H}$):