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Theoretical and numerical analysis of counter-flow parallel convective exchangers considering axial diffusion

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ABSTRACT

We perform a systematic analysis of heat transfer in a counter-current three dimensional convective exchanger, when the inlet/outlet influence is fully taken into account. The analysis, carried out for constant fluid properties, considers the various influences of the fluid/solid conductivity, the imposed convection, inlet/outlet far-field conditions, and lateral boundary conditions. Using a generalized Graetz mode decomposition which permits to consider, both transverse and longitudinal diffusion influence in the exchanger as well as in the inlets/outlets, we put forward several salient generic features of convection/conduction heat transfer.

In all cases we found an optimal Péclet number for the cold or hot effectiveness. Even if, as expected, the larger the Péclet the larger the Nusselt number, high transfer performances are found to be poorly efficient and/or to necessitate non-compact elongated exchangers. Performance degradation arising at high Péclet number are found to be related to "convective leaks" taking place within outlets. A fully developed regime occurs at large Péclet and/or for long exchangers, which is fully determined by the first eigenvalue of the generalized Graetz mode decomposition, which is an extension of classical Graetz analysis. Numerical results are found consistent with a generalized linear relation between effectiveness and the number of heat transfer units asymptotically established in the convection dominated regime. This study opens new perspectives for micro-heat exchangers where moderate convection provides the best effectiveness and compactness. This contribution is also useful for giving reference solutions to counter-flow exchangers with realistic inlet/outlet boundary conditions.

1. Introduction

Conjugate counter-flow heat-exchangers are widely used in thermal and building energy, chemistry, and many other industrial applications [30]. Albeit such exchangers are of low technological content, they support important energetic functions so that proposing new tools to address their optimal use is an important technological issue. In those applied contexts, most of the exchangers are designed with the help of lumped methods, such as the traditional Log Mean Temperature Difference (LMTD) method, compartmental or transverse average approximations, in order to predict and elaborate dedicated look-up tables and graphs for each precise configuration [30,1].

At a more fundamental level, much progress have also been made in the understanding of simple geometry exchangers, e.g. parallel or axi-symmetrical configurations (e.g. [23,18,19,21,22,38,40,39,41,35,36,28] to cite only a few) during the last four decades from studying conjugated Graetz problems. In this specific context, for constant fluids properties, the convection–diffusion problem is linear and amenable to a close solution based on eigenfunction expansions. In counter-flow configurations, these solutions involve sets of real, positive and negative, eigenvalues associated with exponential longitudinal decay, upstream and downstream. Many features of those solutions can be generalized to complex fluids (e.g [29,9]).

More recently, new mathematical analysis have shown that the concept of generalized Graetz modes can be resolved when simultaneously fully taking into account longitudinal conduction and for any general tubular exchanger configuration [26,7,25]. This new framework shows that, indeed, all temperature profiles (except for the very special case of balanced, adiabatic counter-current exchanger) are varying exponentially in the longitudinal direction, in tubular exchangers. This mathematical result is indeed consistent with the LMTD method. In fact, generalized

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Nomenclature

$Bi \equiv \frac{hR}{k_S}$	Biot number
$\mathcal{C}_C^E$	cold base circle in exchanger
$\mathcal{C}_H^E$	hot base circle in exchanger
C^E	external exchanger cylindrical frontier
C_C^E	cold cylinder in exchanger
C_H^E	hot cylinder in exchanger
C_{TI}^C	input cold tube
C_{TI}^H	input hot tube
C_{T0}^C	output cold tube
C_{T0}^H	output hot tube
D_0^C	cold tube disk at $z = 0$
D_0^H	hot tube disk at $z = 0$
D^C	cold tube disk
D^H	hot tube disk
D_L^C	cold tube disk at $z = L$
D_L^H	hot tube disk at $z = L$
$\epsilon_C \equiv \frac{T_C^{-\infty} - T_C^{+\infty}}{T_H^{-\infty} - T_C^{+\infty}}$	cold effectiveness
$\epsilon_H \equiv \frac{T_H^{-\infty} - T_H^{+\infty}}{T_H^{-\infty} - T_C^{+\infty}}$	hot effectiveness
Gr	Graetz number $Gr \equiv Pe/L^\star$
h	convective heat transfer coefficient (Robin/Fourier parameter)
k^F	fluid thermal conductivity
k^S	solid thermal conductivity
L	exchanger length
$L^\star = L/R$	dimensionless exchanger length
$Nu_{H,C}$	Nusselt number in hot/cold tube
$Nu_{H,C}^{loc}$	local Nusselt number in hot/cold tube
$Ntu_H \equiv Nu_H/Gr \equiv Nu_H L^\star/Pe$	number of heat transfer units
$Pe \equiv \frac{\rho c V(2R)}{k^F}$	Péclet number (balanced configuration)
Pe_C	Péclet number in cold tube
Pe_H	Péclet number in hot tube
Pe^o	optimal Péclet number
$Q_{H,C}$	flux in hot or cold tubes
$Q_{H,C}^\star$	dimensionless flux in hot or cold tubes
$Q^\star$	total dimensionless flux
R	tube radius
R_E	exchanger radius
T	temperature field
T_a	reference temperature in the air
$T_C^{-\infty}$	temperature at exchanger input in cold tube
$T_C^{+\infty}$	temperature at exchanger output in cold tube
$T_H^{-\infty}$	temperature at exchanger input in hot tube
$T_H^{+\infty}$	temperature at exchanger output in hot tube
T_w	constant wall temperature

Graetz modes analysis provides a solid theoretical foundation, as well as a clear framework for this known empirical method to be sound. It turns out that the generalized Graetz eigenfunction expansions is also useful to compute complex exchangers properties [25,24] since they permit to map a 3D problem into a 2D one (in the transverse direction). Furthermore, it is important to take into account longitudinal conduction in regions where convective effects are not dominant. Indeed, longitudinal diffusion effects were found significant for Péclet numbers as large as 100 in tubes [33,17]. But this is even more crucial inside the solid domain of an exchanger, since, in the solid part longitudinal and transverse conduction are of similar magnitude. This is especially true in configurations where solid walls are not thin, as opposed to fin exchangers, where a simple thermal resistance model to couple inlet and outlet fluid compartments with solid diffusion is not precise enough.

In the recent years, growing efforts have been dedicated to elaborate micro-heat exchangers for the design of micro-cooling systems associated with high power density micro-chips [3,13,5,6]. Compactness, effectiveness, and performance are indeed alltogether crucial in many contexts and different strategies have been proposed to search for optimal designs (e.g. [34,2,43,11,8]).

Furthermore, in the context of bio-heat transfer modeling of tissue convection within parallel vessels has been considered in many contributions, e.g. [42,44,16,32].

Since, generically, the local transfer rate from the fluid into the solid is found to abruptly decay from the inlet along the longitudinal direction [6,37], a strong emphasis has to be made upon the influence of the inlet and outlet conditions. To be more specific, in many cases a fully developed thermal boundary outlet condition is chosen [39,41,35,36,28,37] to model downstream convection. This is consistent with convective dominated situations with very large Péclet numbers. For more moderate Péclet numbers, a more elaborate coupling of the exchanger with the inlet/outlet is needed to properly take into account the influence of the convection leak inside the outlet, as well as the possible upstream back-conduction in the inlet.

The aim of this contribution is to further explore the influence of inlet/outlet coupling and longitudinal diffusion onto a counter-current exchanger both in the balanced and unbalanced configuration. Whilst it is now easy to perform specific full-3D direct numerical computation with finite volume/finite element/finite differences methods (e.g [6,37]) for a given set of parameters and geometry, it is not yet simple to fully explore the parameter space influence as well as the coupling with inlets/outlets. The latter is especially true since highly convective situations are associated with downstream stretched-out longitudinal temperature gradients inside the outlets, scaling linearly with the Péclet number, so that a full discretization of this compartment is just not worth considering. In this contribution, since we use a generalized Graetz-mode decomposition, all the longitudinal variations are analytically known and do not necessitate any numerical discretization. Solving a two-dimensional numerical problem only and mapping it to a three dimensional one, offers a powerful tool to explore parameter space. Most of the numerical method used in this paper has been detailed in a previous contribution [24]. Hence we will not repeat the method's technical details here in order to lighten the reading of the manuscript. We will rather concentrate on discussing the significance and interest of the obtained results from a physical viewpoint. Most technical aspects related to the mathematical formulation are given in Appendix A.1. The mathematical justification of the method in the context of Robin–Fourier (convective) lateral boundary condition on the exchanger will be worth considering in the future. In this contribution we provide a complete and accurate parameter exploration of counter-current exchangers coupled with their inlet/outlets at very moderate numerical cost, in order to extract the most salient features of the transfer.