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An analytical model for the determination of effective heat conduction of nanofluids

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ABSTRACT

In this study, we use Maxwell's homogenization method followed by mesoscopic analysis to study heat transfer in nanofluids. It is explored how microconvection resulting from the Brownian motion of nanoparticles and the nanolayer formation around nanoparticles are impacting the effective thermal conductivity of nanofluids. The heat transfer within the nanolayer is addressed by direct solution of the Boltzmann transport equation. The results obtained from experimental observations agree with model predictions.

1. Introduction

The thermal conductivity of conventional heat transfer fluids (known as base fluids) including oil, water and ethylene glycol can increase significantly by suspending solid particles (ranging from macro-scale to nano-scale). However, suspension of macro-sized particles are not commercialized yet due to the large size and high density of particles which will tend to settle out of suspension. Emergence of nanotechnology provides new opportunities to produce nanofluids with higher thermal conductivity than the base fluid even in very low concentration of nanoparticles. Note that the term nanofluid refers to suspension of nanoparticles (metallic or nonmetallic) in the base liquid [1–6]. Table 1 shows the bulk thermal conductivities of typical nanoparticles and base fluids. When dealing with heat transfer in nano-scale, conventional macroscopic equations such as the Fourier law breaks down. In other words, the continuum assumption is not longer valid where the characteristic size of the medium is smaller or in the order of the bulk mean free path of the heat carrier. The same scenario is expected to be observed for the effective thermal conductivity of heterogeneous media such as nanocomposites and nanofluids [9–13]. The macroscopic theoretical models such as the Maxwell equation and the Hamilton-Crosser model fail to predict the effective thermal conductivity accurately [9,14,15]. The weakness of these conventional models to explain the enhancement in the effective thermal conductivity of nanofluids indicates some important fundamental physics and mechanisms which are not addressed correctly. It is reported that thermal conductivities of the base fluid and suspended particles in nanofluid are not same as their bulk thermal conductivities due to existence of significant scattering events at interfaces [16,17]. Note that this mechanism is neglected in macroscopic models. However, interface scattering should be somehow taken into consideration in formulating theoretical models of nano-structures. Although various mechanisms and theoretical models have been introduced to explain heat transfer in nanofluids [6,18–20], no comprehensive conclusions have been made. It is pointed out that liquid molecules close to the surface of nanoparticles form a solid like nanolayer [19]. This layer acts as a thermal bridge between the base liquid and nanoparticles resulted in enhancing the effective thermal conductivity. Due to the ordered structure of this nanolayer, its thermal conductivity is supposed to be higher than the thermal conductivity of the base liquid but lower than the thermal conductivity of nanoparticles. Based on the effective medium theory, Yu and Chio developed an expression for the thermal conductivity of nanoparticles with including the nanolayer effects [19]. Xue [21] proposed a formula to predict the effective thermal conductivity of nanofluids by applying the average polarization theory and including the effect of the interface between solid nanoparticles and the base fluid. According to Bruggeman's effective media theory and with including the interfacial shells between the nanoparticles and the base fluids, Xue and Xu [22] obtained an expression for the effective thermal conductivity. In all derivations, the role of interfacial scattering events (i.e. heat carrier-boundary scattering mechanisms) in both nanoparticles and the nanolayer are not taken into account.

There is another important factor that should be considered in thermal analysis of nanofluids. The common assumption in the conventional theoretical models is that the particles are static (motionless) in the host. It is reasonable assumption as long as the host is a solid material or large particles are suspended in the liquid host. When coming to nanoparticles, Brownian motion

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Nomenclature

a_{np}	nanoparticle radius
A	empirical parameter
C	heat capacity
$D(\omega)$	Debye frequency
f	heat carrier distribution function
f_0	heat carrier equilibrium distribution function
h	Plank's constant
k	thermal conductivity
K_B	Boltzmann constant
m	empirical parameter
Pr	Prandtl number
q	heat flux
R	thermal boundary resistance
Re	Reynolds number
s	specularity of the surface
t_{layer}	nanolayer thickness
T	temperature
v	heat carrier velocity

Subscripts

eff	effective
l	base fluid
$layer$	nanolayer
np	nanoparticle

Superscripts

eq	equivalent

Greek letters

α	dimensionless number
ζ	ratio of the thermal conductivity of nanolayer to the nanoparticle
θ	fitting parameter
Λ	heat carrier mean free path
ν	kinetic viscosity
ρ	density
ϕ	volume fraction
ω	frequency

Table 1
Thermal conductivity of various solids and liquids [7,8].

Material	Thermal conductivity [$Wm^{-1}K^{-1}$]
Water	0.6
Ethylene glycol	0.25
Engine oil	0.14
Al_2O_3	36
SiO_2	1.4
Diamond	2300
Silver	429
Copper	401
Aluminium	237

becomes important and implementing additional physics are required to study the effective thermal conductivity [6,18,20,23–25]. It is proposed that the convection caused by the Brownian motion of nanoparticles is the main reason for the enhancement in the effective thermal conductivity of nanofluids. Prasher modified the Maxwell model by incorporating the general correlation for heat transfer coefficient [18]. Jang and Choi [6] proposed a theoretical model that includes four modes; the first mode is interactions between molecules in the base fluid (i.e. physically represents the thermal conductivity of the base fluid), the second mode is the thermal diffusion of nanoparticles in fluids, the collision between nanoparticles is the third mode (neglected) and the last mode is thermal interactions of dynamic or dancing nanoparticles with base fluid molecules. Kumar et al. [24] used the Stokes–Einstein formula to explain the strong dependence of thermal conductivity to the temperature. They proposed a comprehensive model to predict the large enhancement of the thermal conductivity of nanofluids. Bhattacharya et al. [25] used Brownian dynamics simulation technique (which has the advantage of being computationally less expensive than molecular dynamic approach) to compute the effective thermal conductivity of nanofluids. In our study, by a mesoscopic adaptation of the Maxwell equation and by including different physics of nano-scale heat transfer a theoretical model is introduced to predict the effective thermal conductivity of nanofluids. To study the nanolayer effect on the thermal conductivity of nanofluids, the Boltzmann transport equation is directly solved. In previous theoretical approaches, macroscopic models were generally used to analyze the nanolayer effect. Applying our approach will help to study the influence of the boundary scattering (collision between the heat carrier-nanolayer boundary) on the thermal conductivity of nanofluids.

2. Mesoscopic adaptation of the Maxwell model

In this part the mesoscopic adaptation is used to make the Maxwell equation applicable for nanofluids. The Maxwell equation is used as a passage from a heterogeneous to an effective medium. The bulk thermal conductivity of each component is represented in the macroscopic Maxwell model. The formula is only applicable to heterogeneous media in which the particle size is sufficiently large (larger than the heat carrier mean free path). When the size of nanoparticles is smaller than the mean free path of heat carriers, the study has to be conducted in a setting that is more mesoscopic than the setting of the Fourier theory. To determine the material parameters such the thermal conductivity, the mesoscopic theories are entered into calculations. The main problem in this approach is that the microscopic information required as the input of the analysis is usually difficult to acquire and the mathematical formulation is complex. Mesoscopic adaptation of the Maxwell equation is an alternative approach. According to this approach the kinetic theory formula is applied to modify bulk mean free path of each phase in order to take the influence of scattering event into the account. To perform this adaptation, we should find a passage to implement fundamentals of the nanoscale heat transfer into the macroscopic Maxwell model. As presented in the literature [10,26–28], the bulk mean free paths of the base liquid and nanoparticles entering the Maxwell formula should be modified in a way to take interface scattering into account. This approach is called Maxwell's homogenization followed by a mesoscopic analysis. The advantage of this combined approach is that the final outcome is still a close form of the Maxwell equation and complex numerical analysis is not needed. For suspension of spherical particles in a homogeneous liquid, the macroscopic Maxwell equation is written as follows [9,29,30]:

$$k_{eff}^{Maxwell} = k_l \frac{2k_l + k_{np}(1+2\alpha) + 2(k_{np}(1-\alpha) - k_l)\phi}{2k_l + k_{np}(1+2\alpha) - (k_{np}(1-\alpha) - k_l)\phi} \quad (1)$$

The symbol ϕ stands for the volume fraction of nanoparticles, k_{eff} is the Maxwell effective thermal conductivity of the nanofluid, k_l and k_{np} are the heat conductivity coefficients of the base liquid