International Journal of Heat and Mass Transfer 107 (2017) 747–754

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new approach for determining damping factors in Levenberg-Marquardt algorithm for solving an inverse heat conduction problem

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ARTICLE INFO

Article history:
Received 10 October 2016
Accepted 29 November 2016
Available online 7 December 2016

Keywords:
Levenberg-Marquardt algorithm
Damping factor
Inverse heat conduction problem
Temperature-dependent thermal conductivity

ABSTRACT

Damping factor is a key parameter in Levenberg-Marquardt algorithm for solving inverse problems, which significantly affects the efficiency and the convergence stability of Levenberg-Marquardt algorithm and needs to be updated with the iterative number. A new approach is presented for determining damping factors in Levenberg-Marquardt algorithm in this paper, which relates the damping factor directly to the dimensionless objective function in inverse problems. Then, the accuracy, the efficiency, the convergence stability as well as the robustness of Levenberg-Marquardt algorithm are investigated in detail, for identifying temperature-dependent thermal conductivities by solving an inverse heat conduction problem. The results show that the LM algorithm using the new approach for determining damping factors is accurate and robust for identifying temperature-dependent thermal conductivities. The efficiency and the convergence stability of the LM algorithm are improved by using the new approach for determining damping factors. Moreover, the new approach is easy to implement.

1. Introduction

Levenberg-Marquardt (LM) algorithm was proposed by Levenberg [1] and was perfected by Marquardt [2], which originated from solving nonlinear algebraic equations [2]. Besides being investigated for solving nonlinear equations [3–6], LM algorithm has been widely used for solving inverse problems, which have wide range of backgrounds in engineering applications [7–27].

Damping factor is a key parameter [2,28,30,36] in LM algorithm for solving inverse problems, which significantly affects the efficiency and the convergence stability of LM algorithm and needs to be adjusted with iterative numbers. However, there are only a few literatures reporting how damping factor is adjusted with iterative numbers, although LM algorithm is frequently employed for solutions of inverse problems [28–38]. Marquardt [2] gave a general method for determining the damping factor in LM algorithm and it has been widely employed [32]. Ukrainczyk [28] presented an improved apparatus and LM algorithm to obtain the estimate of thermal diffusivity, and gave a method for determining damping factors. LM algorithm was employed for solving the inverse boundary design problem of natural convection-radiation in a square enclosure, and the damping factor is constant and chosen as 0.2 [30]. Kind et al. [36] used macro-indentation and LM algorithm for identification of plastic properties, and they found that the minimum damping factor was 2, and the maximum damping factor was 10, to guarantee a convergence. Damping factors were also investigated for solving nonlinear equations, and several methods were suggested [3–5]. Fan and Yuan [3] proposed that damping factor was the power of nonlinear functions' 2-norm, and the power was between 1 and 2. In Refs. [4] and [5], the methods for determining damping factors are often impractical, when LM algorithm is employed for solving inverse problems. For example, nonlinear equations are directly equal to zero. Differently, solving inverse problems is an optimization process, minimizing the objective function, and the objective function is always non-zero in practical applications. For an inversion process, the first order derivatives of the objective function with respect to inverted parameters should be equal to zero, which make up of nonlinear equations. For solving these nonlinear equations, the first order derivatives of the equations are usually required for determining damping factors in LM algorithm [4,5], and it means that the second order derivatives of the objective function with respect to inverted parameters in inverse problems have to be determined. This is rather complicated and even impossible in most circumstances.

Recently, the first author and co-authors have presented a modified LM algorithm for estimating boundary heat flux in Ref. [37], and sensitivity coefficients are accurately determined by using the complex-variable-differentiation method (CVDM) [39]. In the

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.11.101
0017-9310/

Nomenclature

c	heat capacity, J/(kg °C)
E_{rms}	percentage relative error between the recovered/inverted and the exact/real values
M	total number of measured temperatures
N	total number of inverted parameters
n	power
P	the iteration number
q	heat flux, W/m^2
S	objective function
t	temperature, °C
v	parameter in Marquardt's method
W	the width of the modeled object in y-direction, m
Y	vector of inverted parameters
y	y-coordinate, m

Greek

γ	exact value
δ	updated vector of recovered/inverted parameters
ζ	random measurement error
η	random number
λ	thermal conductivity, W/(m °C)
ξ	small positive number
ρ	density, kg/m^3
χ	inverted value
τ	time, s

Subscripts

0	initial time
b	bottom
exact	exact
i	the ith component of a vector
k	the kth component of a vector
u	upper

Superscripts

0	initial guess
*	measured

previous work [37], the damping factor was determined by using Marquardt's method [2], and good performances were obtained for recovering boundary heat flux. However, divergences frequently occur when LM algorithm is employed for identifying multi temperature-dependent thermal conductivities by solving an inverse heat conduction problem, using Marquardt's method [2] or Ukrainczyk's method [28] for determining damping factors. Meanwhile, the efficiency is very slow using Fan and Yuan's method [3] for determining damping factors in LM algorithm. Therefore, it is necessary to propose a new approach for determining damping factors in LM algorithm, for accurately, efficiently, and stably identifying multi temperature-dependent thermal conductivities. This is the motivation of the present work originates from.

In this paper, a new approach is proposed for updating damping factors with iterative numbers, in which the damping factor is directly related to the dimensionless objective function in an inverse problem. Then, the accuracy, the efficiency, the convergence stability, and the robustness of LM algorithm using the new approach for determining damping factors are investigated in detail, and compared with that of using other methods for determining damping factors, for identifying temperature-dependent thermal conductivities by solving an inverse heat conduction problem.

2. Transient nonlinear heat conduction problem

In the present work, the thermophysical properties are only temperature-dependent and isotropic, i.e., they are nonlinear and independent on dimensions. The source term and the phase change are not involved. Then, the transient nonlinear heat conduction problem can be expressed as follows:

$$\rho(t)c(t)\frac{\partial t(y)}{\partial \tau} = \frac{\partial}{\partial y}\left[\lambda(t)\frac{\partial t(y)}{\partial y}\right] \tag{1}$$

with the initial condition given in Eq. (2).

$$t(y,\tau)|_{\tau=0} = t(y) \tag{2}$$

The hybrid boundary conditions are given in Eqs. (3) and (4).

$$t(y,\tau)|_{y=W} = t_u(\tau) \tag{3}$$

$$\lambda(t)\frac{\partial t(y)}{\partial y}|_{y=0} = q_b(\tau) \tag{4}$$

In Eqs. (1)-(4), t is the temperature, ρ is the density, c is the mass specific heat, λ is the thermal conductivity, y is the coordinate along the width of the object, W is the width of the object, τ is time, q is heat flux, and subscripts u and b represent upper and bottom boundaries, respectively.

The objective of solving the transient nonlinear heat conduction problem is to determine transient temperatures at measurement points for inverse analysis, and the finite difference method derived in Ref. [15] is employed for the solution.

3. Levenberg-Marquardt algorithm [37] for an inverse heat conduction problem

In the inverse problem, temperature-dependent thermal conductivities are unknown and need to be identified, but everything else are kept the same as that in the heat conduction problem. Additional temperature measurements are required, which are simulated by solving the heat conduction problem. In engineering applications, these additional temperatures are usually from measurements. The inverse problem can be constructed as a problem of minimization of the following objective function:

$$S(Y) = \sum_{i=1}^{M}(t_i(Y) - t_i^*)^2 \tag{5}$$

In Eq. (5), M is the number of temperature measurements, Y=(Y_1, Y_2,. . ., Y_N) is the vector of the inverted thermal conductivities, N is the number of parameters to be inverted, Y_i is the ith inverted parameter. t_i^* and t_i are the measured and calculated temperatures, respectively, i = 1, 2, . . ., M.

In LM algorithm, the inverted thermal conductivity vector is updated by Eq. (6).

$$Y_k^{P+1} = Y_k^P + \delta^P \tag{6}$$

In Eq. (6), P is the iteration number, k = 1, 2, . . ., N, and δ is determined by solving Eq. (7).

$$[J^TJ + \mu\, diag(J^TJ)]\delta = J^T[t_i^* - t_i(Y)] \tag{7}$$