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Heat pump model for Ranque–Hilsch vortex tubes

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ABSTRACT

We describe a quantitative model for heat separation in a fluid due to motion along a pressure gradient. The physical model involved is relevant to one explanation for the temperature separation in a vortex tube. This effect has a point of saturation in which the fluid's temperature and pressure are related at its boundaries by an adiabatic law. Vortex tube models sometimes assume that this saturation is achieved in physical devices. We conclude that this is likely to be a safe assumption much of the time, but we describe circumstances in which saturation might not be achieved. We propose a test of our model of temperature separation.

1. Introduction

A Ranque–Hilsch vortex tube is a device which separates initially constant-temperature gas into hot and cold streams. A typical vortex tube consists of a cylindrical container into which gas is injected in a direction perpendicular to the cylinder axis. The injected gas sets up a rapidly rotating flow inside the tube. Gas is allowed to exit the tube from either end, but the exits of each end are configured so that one end primarily draws gas from the outer edge of the cylinder, whereas the other end primarily lets out gas from the central region. The Ranque–Hilsch vortex tube is named for Ranque, who invented the device, and Hilsch, who made important early contributions to its study [1,2]. Vortex tubes are used in industry for a variety of applications, generally involving spot cooling.

Since the discovery of vortex tubes, there has been debate and discussion about the details of how the characteristic temperature separation comes about. Some early theories suggested that the effect was caused by friction between concentric annular regions of the rotating tube [2]. Deissler and Perlmutter performed an analysis [3] which suggested that turbulent shear work was the most important cause of the temperature separation. Kurosaka developed a theory [4] that explained the temperature separation in terms of acoustic streaming. Stephan et al. suggested [5] that Görtler vortices on the walls of the tube were an important factor.

One general approach, which has been used by several authors, seeks to explain the temperature separation effect in terms of adiabatic heating and cooling. The rotating flow sets up a radial pressure gradient to balance the centrifugal potential. If some fluid moves radially back and forth, it will tend to be adiabatically heated as it moves from the core to the periphery and adiabatically cooled as it moves from the periphery to the core. Kassner and Knoernschild [6] introduced the premise that radial motion will make the temperature distribution follow an adiabatic law, that is,

$$T(r) \sim p(r)^{(\gamma-1)/\gamma}. \quad (1)$$

Their explanation involved an initially irrotational vortex (that is, with angular velocity that scales with radius like r^{-2}) which converts to a rotational vortex (with constant angular velocity) due to effects at the outer boundary and the core. They suggest that the radial motion which brings about this temperature distribution will be driven by turbulence.

Ahlborn and Groves have reported experimental observations [7] of a secondary flow in a vortex tube, which includes both axial and radial motion. Their measurements did not determine whether this secondary flow was open or closed. Subsequently, Ahlborn and collaborators suggested a model in which this secondary flow played a key role [8,9]. In this model, the secondary flow sets up a refrigeration cycle in the vortex tube.

Computational fluid dynamics work carried out by Behera et al. [10] confirmed the existence of such a secondary flow for vortex tubes in which the cold end diameter was small compared to the diameter of the rest of the tube. However, they reported that it disappears for vortex tubes in which the cold end diameter was less small. Behera et al. found that the simulations with smaller cold end diameters more closely matched the model put forth by Ahlborn et al., but that the simulations with larger cold end diameters did not closely match that model and had substantially larger temperature separations. In general, computational studies have

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played an important role in vortex tube research, addressing a wide variety of issues involving vortex tube flow and temperature separation [11–15].

Liew et al. made a quantitative model using the adiabatic law given in Eq. (1) and relying on the presence of radial motion within the vortex tube [16]. In order to predict the temperatures measured at the two exits of the tube, they also included a term to account for adiabatic deceleration as the fluid moves axially toward the hot-stream exit.

In this paper, we explore a simple analytical model of a vortex-tube-like heat pump relying on adiabatic heating and cooling of fluid moving along a pressure gradient. We will use this model to try to address some of the questions and assumptions involved in existing vortex tube models. In particular, if a subset of a fluid is moving up and down a pressure gradient, how reasonable is it to assume that the entire system will attain the temperature distribution described in Eq. (1)? Furthermore, to what extent does it matter what the radial flow looks like — that is, if the flow is due to an open secondary flow, a closed secondary flow, or a more general kind of radial mixing?

2. The heat pump

To construct a quantitative model for heat transport, suppose a parcel of gas is being moved up and down a potential gradient. A schematic of the cycle is shown in Fig. 1. For simplicity, consider a gravitational potential so that the pressure is higher at smaller z. In practice, any pressure gradient between $z = 0$ and $z = L$ produces the same effect. In a vortex tube, the centrifugal potential produces the pressure gradient.

Broadly speaking, our model involves parcels of gas whose motion along a pressure gradient causes them to be adiabatically cooled and heated. While they are cooled (or heated), they do work on the surrounding medium (or the other way around), which facilitates energy transfer between the parcel and its surroundings. However, this is not enough to establish a temperature gradient in the surrounding fluid. If the work associated with adiabatic heating and cooling were the only source of energy transfer, then the work done during the cooling step would exactly cancel with the work received in the heating step, and no net energy would be exchanged between the moving parcels and the surrounding fluid. For this reason, we include steps after each adiabatic heating or cooling step during which the moving parcel is allowed to exchange heat with its surroundings, either by conduction or by mixing.

The parcel of gas starts at some position $z = L$ and internal temperature $\overline{T}_1$ then moves to a new position $z = 0$. Suppose the background medium has some pressure distribution $p(z)$ with $p_0 \equiv p(0)$ and $p_L \equiv p(L)$. Then we can define a dimensionless constant

$$\alpha \equiv \left(\frac{p_0}{p_L}\right)^{(\gamma-1)/\gamma}. \tag{2}$$

γ denotes the specific heat ratio c_p/c_v. We will assume that the moving parcel is always in pressure equilibrium with its surroundings (the pressure equilibration time should be much shorter than the temperature equilibration time). If the motion of the fluid parcel is adiabatic, then the new temperature will be $\overline{T}_2 = \alpha\overline{T}_1$. Then suppose the fluid element remains at $z = 0$ for some period of time, during which it exchanges heat with the surrounding medium. If the surrounding medium has some temperature T_0 at $z = 0$, then the temperature of the gas in the parcel during this time will satisfy the heat transfer equation

$$\frac{d\overline{T}}{dt} = \frac{-Ah}{c_p N_p}\left(\overline{T}_2 - T_0\right). \tag{3}$$

Here A is the area of the interface between the parcel of gas and the environment, h is the heat transfer coefficient, c_p is the constant-pressure heat capacity, and N_p is the number of particles in the parcel. We will assume that the temperature inside the parcel is homogeneous. After heat has been exchanged for an interval τ_e, we get a new temperature. If the heat exchange is entirely due to heat conduction, that new temperature will be

$$\overline{T}_3 = T_0 + \left(\overline{T}_2 - T_0\right)\exp\left(\frac{-Ah\tau_e}{c_p N_p}\right). \tag{4}$$

If the fluid parcel then adiabatically rises back to $z = L$, the new temperature will be $\overline{T}_4 = \overline{T}_3/\alpha$. Finally, if the parcel once again exchanges heat isobarically with the surrounding environment before the cycle begins again, and if the background medium has temperature T_L at $z = L$, we have

$$\overline{T}_1 = T_L + \left(\overline{T}_4 - T_L\right)\exp\left(\frac{-Ah\tau_e}{c_p N_p}\right). \tag{5}$$

We will assume for the sake of simplicity that the exponential factor $-Ah\tau_e/c_p N_p$ is the same for both points in the cycle where the parcel undergoes isobaric heat exchange (realistically, several of those terms might vary, but the overall behavior of the system should not change too much). We will define this factor by

$$r \equiv \exp\left(\frac{-Ah\tau_e}{c_p N_p}\right). \tag{6}$$

If $r = 1$, then there is no transfer of energy during these steps of the cycle and the adiabatic compression and expansion steps will cancel each other out. If $r = 0$, then the parcel of moving gas achieves full thermal equilibrium with its surroundings during the heat conduction steps.

If the parcel returns to the same temperature $\overline{T}_1$ with each cycle, then it follows that $\overline{T}_1$, $\overline{T}_2$, $\overline{T}_3$, and $\overline{T}_4$ can each be expressed in terms of the temperature of the background fluid. For instance,

$$\overline{T}_1 = \frac{\alpha T_L + rT_0}{\alpha(1+r)}. \tag{7}$$

We can use this information to solve for the total heat transferred between the parcel and the surrounding fluid during the second and fourth steps. They are

$$Q_{\text{bot}} = \frac{c_v N_p (1-r)}{1+r}(\alpha T_L - T_0) \tag{8}$$

$$Q_{\text{top}} = -\frac{c_v N_p (1-r)}{\alpha(1+r)}(\alpha T_L - T_0). \tag{9}$$

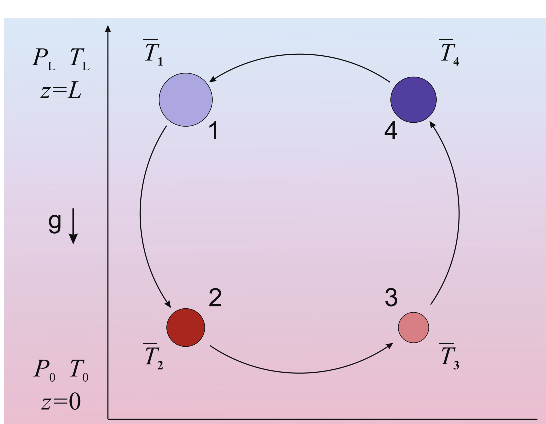

Fig. 1. This diagram shows the heat transfer cycle used in our model.