International Journal of Heat and Mass Transfer xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Variational multiscale element free Galerkin method for natural convection with porous medium flow problems

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ARTICLE INFO

Article history:
Received 14 July 2016
Received in revised form 28 October 2016
Accepted 2 November 2016
Available online xxxx

Keywords:
Variational multiscale method
Meshless method
Porous medium flow
Natural convection problems

ABSTRACT

In this paper, the variational multiscale element free Galerkin (VMEFG) method is extended to simulate the two-dimensional natural convection with porous medium flow problems. The Darcy-Brinkman-Forchheimer extended model is used for the non-Darcy porous medium flow. The VMEFG method is a meshless method which had successfully applied to simulate incompressible flow problems with equal order basis approximation for the velocity and pressure fields, meanwhile, this method can easily deal with complex geometry problems. Three numerical examples, including the natural convection in a square cavity, a cavity with cosine vertical wavy wall and a triangular cavity with zig-zag shaped bottom are considered to investigate the validation and accuracy of the proposed method. The numerical results are presented in the form of streamlines, isotherms and average Nusselt numbers. A good agreement is observed between the present results and those available in the previous studies. Meanwhile, effects of Rayleigh number, porosity of the porous medium, amplitude of wavy surface and number of undulation on the flow structure and heat transfer are also investigated.

1. Introduction

The natural convection with porous medium flow problems are a scientific subject which has been investigated for many years. Due to its importance in many geophysical and engineering applications such as regenerative heat exchangers, electronic cooling, geothermal energy system, thermal insulation and so on, the analysis of natural convection in porous medium is very important in science and engineering. The natural convection with porous medium flow problem has a highly nonlinearity, thus, analytical solution to this problem is difficult to obtain and usually solved by numerical method. To date, the natural convection in porous medium has been extensively investigated by many researchers through all kinds of numerical methods. For example, Satya et al. [1] applied finite element method (FEM) to investigate the natural convection with porous medium in Darcy and non-Darcy regimes. Nithiarasu et al. [2] used a semi-implicit characteristic-based split algorithm with FEM to solve the natural convection in a square cavity with non-Darcy porous medium. Misirlioglu et al. [3] also adopted FEM to investigate the natural convection with Darcy porous medium in the wavy cavity. Chen et al. [4] studied the free convection in a wavy cavity with Darcy-Brinkman-Forchheimer extended model porous medium by finite volume method (FVM). Khanafer et al. [5] applied FEM to analyze the natural convection inside a cavity with a sinusoidal vertical wavy wall and filled with a non-Darcy porous medium. Zhao et al. [6] proposed lattice Boltzmann method (LBM) to simulate the natural convection flow in a square cavity filled with porous medium.

Recently meshless methods have been developed as a new tool for simulating the fluid flow, heat transfer and other complicated physical phenomena. Compared with the traditional computational fluid dynamics methods based on mesh, the meshless methods are independent of mesh and require only a set of nodes scattered within the problem domain and its boundary in which this is an important and attractive advantage. At present, there exist many kinds of meshless method. The more details about these meshless methods and their applications can refer to some review articles [7,8] and books [9–11]. However, there are a few implementations of these meshless methods in the fluid flow and convection heat transfer problems with in porous medium. Taking meshless methods based on Galerkin form as an example, there are two main difficulties about simulation of Navier-Stokes equations to work out or overcome, that is, velocity oscillations for convection dominated and pressure spurious oscillations for incompressibility [12,13]. Up to now, a few stabilization methods have been proposed to resolve or overcome numerical spurious oscillations for meshless methods. In general, the idea of these stabilization

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.11.008
0017-9310/© 2016 Published by Elsevier Ltd.

methods is evolved from the FEM, such as streamline-upwind Petrov-Galerkin (SUPG), Galerkin least-squares (GLS), Pressure-stabilizing Petrov-Galerkin (PSPG), projection or the fractional step method (e.g. characteristic-based split algorithm) and so on. However, the stabilization parameter of these methods is usually obtained depending on the problem under consideration and the selected numerical method. To overcome this shortcoming, Zhang et al. [14] proposed the variational multiscale element free Galerkin method to solve the Stokes flow problems. VMEFG method coupled the element free Galerkin method and variational multiscale method which first proposed by Hughes et al. [15], therefore, it inherits the advantages of variational multiscale method and meshless methods. For example, VMEFG method allows equal order basis approximation for pressure and velocity, which is easy to implement in the viewpoint of application. Meanwhile, VMEFG method is free of user-defined stabilization parameter because it appears naturally during the derivation process. So far, VMEFG method has successfully solved the Stokes equations [14], convection-diffusion-reaction equations [16], water wave problems [17], magnetohydrodynamics (MHD) flow problems [18], non-Newtonian flow problems [19], natural convection problems [13,20].

As a matter of fact, there are a few reports on application of meshless methods or mesh reduction methods in natural convection with porous medium flow problems. Šarler et al. [21] firstly attempted at solving the problem of Darcy natural convection in porous medium by the radial basis function (RBF) collocation. Subsequently, Perko et al. [22] also applied RBF collocation to solve the natural convection in porous medium for the Darcy-Brinkman model. Kosec et al. [23] explored the application of local RBF collocation method in solution of natural convection problem in Darcy porous medium. Fan et al. [24] attempted to analyze the double-diffusive natural convection in paralleogrammic enclosures filled with fluid-saturated porous media by local RBF collocation method. In their analysis, they only considered the Darcy model. Bourantas et al. [25] adopted a meshless point collocation method which used the moving least squares approximation for the field variables to investigate the natural convection of a nanofluid with Darcy-Brinkman porous medium. Šarler et al. [26] used dual reciprocity boundary element method (DRBEM) to simulate the natural convection in Darcy-Brinkman porous medium in terms of primitive variables. Recently, Pekmen et al. [27] also used DRBEM to solve Brinkman-Forchheimer-Darcy extended model in a porous medium, but their formulations were based on stream function, vorticity and temperature iteratively.

From above brief review, we can note that most of numerical studies were mainly focused on natural convection problem with simple geometry, such as rectangular geometry, and almost no meshless method research of natural convection with porous medium problems in complex enclosures was discussed. Although many numerical schemes are adopted to successfully analyze the natural convection with porous medium flow problem, it is still necessary to develop a stable and easy-to-use numerical algorithm for solving this challenging problem. Thus, in the paper, the VMEFG method is used to solve Darcy-Brinkman-Forchheimer extended model in porous medium with complex geometry. To the best of the authors' knowledge, this model is not solved by any of Galerkin-based meshless method.

An outline of the paper is as follows: in Section 2 the governing equations of Darcy-Brinkman-Forchheimer extended model in porous medium flow are briefly presented. In Section 3 the approximation method of moving least squares (MLS) is simply described, and then the implementation of the VMEFG method to simulate natural convection problems with porous medium is expressed. In Section 4 the numerical results and discussion are presented and conclusions are drawn in Section 5.

2. Mathematical model

The flow is assumed to be incompressible, laminar and two-dimensional. The porous medium is considered to be rigid, homogeneous, isotropic and saturated with the same single-phase fluid. The generalized Darcy-Brinkman-Forchheimer model [2–5] is adopted to model the flow in the porous medium, where the convective, viscous, and inertial effects are considered. The dimensionless forms of governing equations can be written as:

Continuity equation

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

momentum equation

$$\frac{1}{\varepsilon^2}(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{Pr}{\varepsilon}\nabla^2 \boldsymbol{u} - F\boldsymbol{u} + RaPrT\boldsymbol{j} \tag{2}$$

where

$$F = \frac{Pr}{Da} + \frac{1.75}{\sqrt{150}}\frac{|\boldsymbol{u}|}{\sqrt{Da}}\frac{1}{\varepsilon^{3/2}} \tag{3}$$

energy equation

$$\boldsymbol{u} \cdot \nabla T - \nabla^2 T = 0 \tag{4}$$

where ε is the porosity of the porous medium, $\boldsymbol{u} = [u, v]^{\mathrm{T}}$ is the dimensionless velocity vector, $\boldsymbol{j} = [0, 1]^{\mathrm{T}}$ is the unit vector in the direction of the gravitational force; p is the dimensionless local average pressure; T denotes the dimensionless temperature; Pr is the Prandtl number; Ra is the Rayleigh number and Da is the Darcy number.

The local Nusselt number along the hot wall is calculated as

$$Nu = \frac{\partial T}{\partial \boldsymbol{n}} \tag{5}$$

where $\boldsymbol{n}$ denotes the outward normal to the block surface. The surface average Nusselt number is calculated by integration of local Nusselt number over the wall as

$$Nu_{avg} = \frac{1}{S}\int_0^S Nu ds \tag{6}$$

Here S is the length of the wall where the local Nusselt number is evaluated.

3. The VMEFG method

For the incompressible flow, the VMS method has already been proposed by Masud et al. [28] in the context of the FEM. Here, we will extend VMS in the context of the element free Galerkin (EFG) method for the porous medium flow problems. The biggest difference of EFG method and FEM is the construction of shape function. For the EFG method, the moving least squares approximation (MLS) is used to construct meshless shape function and is briefly described in the following.

3.1. The fundamental of MLS

This section has mainly been taken from [13,20]. In the MLS, the approximation of field variable $f(\boldsymbol{x})$ mainly consists of three parts: a basis function, a group of nonconstant coefficients and a weight function associated with each node. According to the MLS, the approximation function $f^h(\boldsymbol{x})$ of field variable $f(\boldsymbol{x})$ is defined by:

$$f^h(\boldsymbol{x}) = \sum_{j=1}^{m} p_j(\boldsymbol{x})a_j(\boldsymbol{x}) = \boldsymbol{P}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{a}(\boldsymbol{x}) \tag{7}$$

where $\boldsymbol{P}(\boldsymbol{x})$ is a polynomial basis function of the spatial coordinates, m is the number of monomial terms in the basis function, and $\boldsymbol{a}(\boldsymbol{x})$

Please cite this article in press as: J. Chen et al., Variational multiscale element free Galerkin method for natural convection with porous medium flow problems, Int. J. Heat Mass Transfer (2016), http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.11.008