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An MHD couple stress fluid due to a perforated sheet undergoing linear stretching with heat transfer

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ABSTRACT

We investigate an MHD couple stress liquid due to a perforated sheet undergoing linear stretching with radiation. The liquid is initially at rest with its activity is restricted by pulling the two sheet ends with parallel and identical forces. The consequential movement of the or else quiescent fluid is consequently generated exclusively by the stirring plate that develops a linearly varied speed with the distance from the slit. In addition to fluid flow, heat transfer with two cases of different boundary conditions from the sheet is considered, the first with prescribed surface temperature and, the second with prescribed heat flux. The arising set of non-linear coupled nonlinear partial differential equations is rehabilitated into non-linear ordinary differential equations and then exact expressions are derived for velocity and by means of a power series method with Kummer's confluent hyper-geometric functions for temperature.

1. Introduction

The theoretical fluid mechanics phenomena involving steady/unsteady laminar boundary layers be of large hypothetical furthermore convenient importance (see Schlichting [19]). The present problem has wide physical application in extrusion impulsively/linearly/non-linear stretching process like condensation process of aerodynamic development, cooling method of tinny sheet and in the glass, polymers and there are also several applications in bio-fluid dynamics, aeronautics and manufacturing (see Fisher [8]). Couple stress liquids over a linear stretching sheet has established significant concentration since of their widespread applications in the field of metallurgy. The developed of polymer fibers, by means of the dissolve whirling procedure, involves the extrusion of molten fibre through an orifice. Similar flows involving magnetic fields (magnetohydrodynamic flow) are also extremely vital technologically and applications in special areas of interest such as petroleum production and metallurgical processes can be found. Together to the flow, it is found also that the properties of the end products depend strongly lying on the speed of cooling implicated in these processes. Magnetic fields have been used already in the process of refinement of molten metals from nonmetallic inclusions, and Sarpakaya [20] was the initial examiner to investigate the MHD flows of a non-Newtonian liquid.

A widespread variety of mathematical models has been developed to reproduce the different hydrodynamic/hydromagnetic behavior of these non-Newtonian liquids. Expressive expositions of viscoelastic fluid models have been investigated by Joseph [10]. Examples of such models are the Rivlin–Ericksen second order model [15,16] and Siddappa and Khapate [22] and the Oldroyd model [32] see also Bhatnagar [3] where the flow of an Oldroyd-B liquid is considered, occupying the space over an elastic plate, due to the stretching of the sheet in the presence of a constant free-stream velocity. Moreover, the Johnson–Seagalman model, the upper convected Maxwell model, see Rao [26,27], Rao and Rajgopal [28] and the Walters' liquid B model [31], Siddheshwar and Mahabaleshwar [21] and Mastroberardino and Mahabaleshwar [13] have been proposed. Recently, Maxwell model and Oldroyd-B model have been used to study the flow of viscoelastic liquids on top of stretching and non stretching sheets but with no heat transfer effects involved. Together steady and unsteady flows have been presented expansively in a different variety of geometries using a wide spectrum of analytical, semi-analytical and numerical methods (see Mahabaleshwar [14],

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Nomenclature

A, B, D	constants
C_p	constant pressure [W kg^{-1} K^{-1}]
c	stretching rate [s^{-1}]
C	$\left(=\frac{c\eta_0}{\rho\nu^2}\right)$ couple stress parameter
C_1	streamline constants
f	similarity variable
$_1F_1$	(=F[a, b, z]) Kummers' expression
$_2F_1$	(=F[a, b, c, z]) Kummers' expression
g	$\left(=\frac{T-T_\infty}{T_w-T_\infty}\right)$ for the PHF case
H_0	magnetic field [w m^{-2}]
k	conductivity [W kg^{-1} K^{-1}]
k^*	mean absorption coefficient [m^{-2}]
l	length [m]
M	$\left(=H_0\sqrt{\frac{\sigma}{c\rho}}\right)$ Hartmann number also called Chandrasekhar number
N_I	$\left(=\frac{Q^*}{c\rho C_p}\right)$ heat source/sink parameter
N_R	$\left(=\frac{16\sigma^* T_\infty^3}{3kk^*}\right)$ radiation number
Nu_x	$\left(=\frac{xq_w}{k(T_w-T_\infty)}\right)$ Nusselt number
Pr	$\left(=\frac{\nu}{\alpha}\right)$ Prandtl number
q_r	radiative heat flux [W kg^{-1} m^{-1}]
q_w	local heat flux at the wall
Q	$(=M^2)$ Chandrasekhar number
Q_s	heat source [W kg^{-1} K^{-1} m^{-2}]
Re_x	$\left(=\frac{xU_w}{\nu}\right)$ local Reynolds number
Rm	$\left(=\frac{cl^2}{\nu_m}\right)$ magnetic Reynolds number
s	wall temperature parameter
T	temperature [K]
$T\infty$	far away from the sheet [K]
T_w	wall (sheet) temperature [K]
U_w	stretching velocity of the sheet
u	velocity component beside the sheet [m s^{-1}]
v	velocity component normal to the sheet [m s^{-1}]
x	x-coordinate along the sheet [m]
y	y-coordinate normal to the sheet [m]

Greek symbols

α	$\left(=\frac{k}{\rho C_p}\right)$ thermal diffusivity [m^2 s^{-1}]
δ	thickness of boundary layer
δ_η	$\left(=\delta_{\eta_1}\sqrt{\frac{\nu}{c}}\right)$ velocity boundary layer thickness
δ_T	$\left(=\delta_{T_1}\sqrt{\frac{\nu}{c}}\right)$ thickness of thermal boundary layer
δ_{η_1}	dimensionless velocity boundary layer thickness
δ_{T_1}	dimensionless thermal boundary layer thickness
η	similarity variable
η_0	material constant for the couple stress fluid
μ	limiting viscosity [kg m^{-1} s^{-1}]
ν	$\left(=\frac{\mu}{\rho}\right)$ kinematic viscosity [m^2 s^{-1}]
ν_m	magnetic permeability
ν'	$\left(=\frac{\eta_0}{\rho}\right)$ couple stress viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
ψ	stream function [m^2 s^{-1}]
σ	electrical conductivity [mho m^{-1}]
σ'	Stefan–Boltzmann constant
ξ	change of variable
τ_w	wall shearing stress [m^2 s^{-1}]
θ	$\left(=\frac{T-T_\infty}{T_w-T_\infty}\right)$ for the PST case

Siddheshwar and Mahabaleshwar [21], Andersson et al. [2], Turkyilmazoglu and Pop [25] and Xu et al. [33]).

The theory of Boussinesq- Stokes suspension that displays the effect of couple stress and the constitutive equations for couple stress liquids is due to Stokes (1966). A imperative group of pupils of non-Newtonian liquid model is the couple stress replica which is a vigorous models for definite polymeric materials. Sakiadis [15–18] was the first researcher that discusses the boundary layer flow theoretically, numerically and experimentally and then his theory was extensive further by Crane [3]. He pointed out that in the polymer industry it is sometimes essential to consider a stretching plate. An analytical form was presented by Crane [3]. Crane flow was investigated by Gupta and Gupta [11] for the heat and mass transfer over a linear stretching plate in the being there of suction/injection issuing from a thin slit. A non-isothermal moving sheet was dealt with and the temperature and concentration distribution profiles for that situation were obtained. Carragher and Crane [5] analyzed the heat transfer owing to a continuous stretching plate. Pavlov [26,27] obtainable an correct resemblance explanation of the magnetohydrodynamics. The heat and mass transfer over a stretching sheet with or without suction/blowing and with/without magnetic field is studied by other researchers (Fox et al. [9], Chen and Char [6], Tsou et al. [24], Mahabaleshwar [14], etc.) by taking different situations.

All the above mentioned researchers restricted their analyses to viscous flows. Many unexplored aspects of the linear stretching sheet problem are unearthed by the present study which is a overview of the works of Crane [7] and Pavlov [26]. Motivated by every person these investigates we intend to examine an MHD couple stress fluid due to a perforated sheet undergoing linear stretching with heat transfer. Furthermore for MHD flows, the effects of heat source/sink, radiation, wall temperature and magnetic field on couple stress fluid flow over a stretching sheet are discussed. Presents consequences have potential scientific applications in fluid based systems concerning stretchable supplies, in polymer extrusion process and similar problems.

2. Mathematical formulation and solution

Let us consider an MHD couple stress liquid due to a perforated sheet undergoing linear stretching with radiation. The two-dimensional flow of the fluid is confined to the half space $y > 0$ above the sheet as shown in Fig. 1. The plate is being

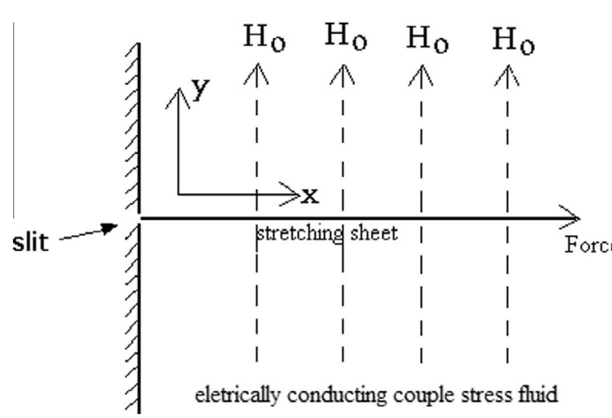

Fig. 1. Schematic of the two-dimensional stretching sheet problem.